

# RG Models Used in TFT

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- **Crow-AMSAA**
- **Duane**
- Gompertz and Modified Gompertz
- Lloyd Lipow
- Logistic

# Duane Model

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- Also called as “Duane Postulate”.
- Developed based on an observation by J.T. Duane.
- Duane observed that the cumulative failure rate vs cumulative test time was linear on a log-log paper when he plotted test data from generators, hydro-mechanical devices and aircraft engines.
- It is a **deterministic model**.
- Used for Test-Fix-Test data.

# Duane Growth Model

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$$MTBF_C = MTBF_0 \left( \frac{T}{T_0} \right)^\alpha$$

- $MTBF_C$  is cumulative=total failures/total time.
- $MTBF_0$  is at starting point.
- $T_0$  is starting time and  $\alpha$  is the growth rate.
- Can be used to estimate required test time to reach goals or if time is fixed, determine how many systems must be tested .
- With good effort  $\alpha \sim 0.3-0.4$  for complex systems.
- Instantaneous  $MTBF_i = MTBF / (1 - \alpha)$ .

# Meaning of Alpha Values

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- The slope, alpha, gives an indication of the rate of MTBF growth.
- Shows effectiveness of the company's reliability program.
- Duane observed that alpha ranged between 0.2 and 0.4 and that range of values was a very good correlation with the intensity of the effort on the reliability growth program.
- The higher the alpha value, the bigger the effects of the improvements (reliability growth) in the reliability of the item.
- When  $\alpha < 0$  or  $\alpha = 0$ , there is NO Growth.

# Duane Growth Model – Example

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$$MTBF_C = MTBF_0 \left( \frac{T}{T_0} \right)^\alpha$$

- Let's see how this function looks over time with a range of growth rates.
- Let's look at the following cases and plot
- $MTBF_C / MTBF_0$  vs.  $T/T_0$ 
  - $\alpha = -0.075$
  - $\alpha = 0.0$
  - $\alpha = 0.3$
  - $\alpha = 1.0$

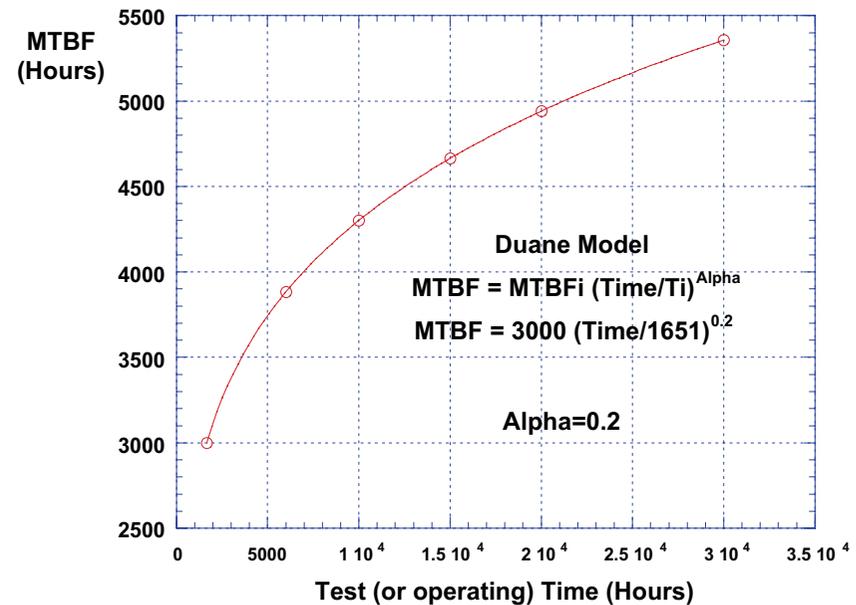
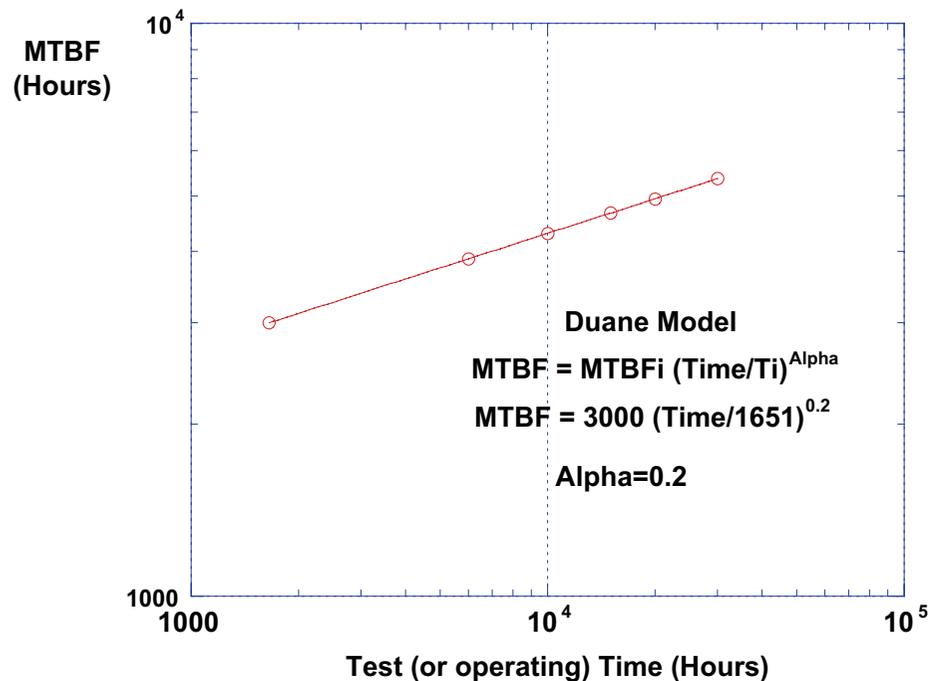
# Duane Reliability Growth Model

- Duane obtained a straight line on a log - log plot of MTBF versus operating time

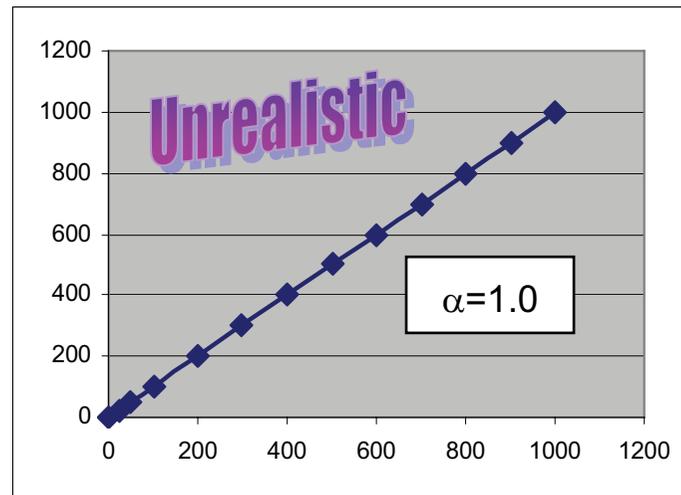
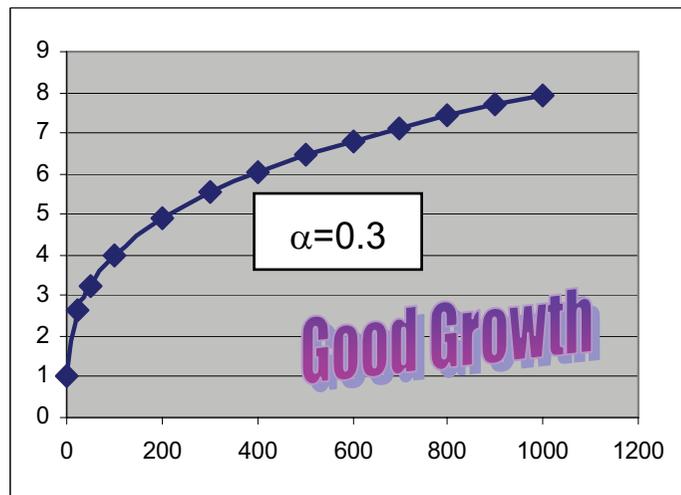
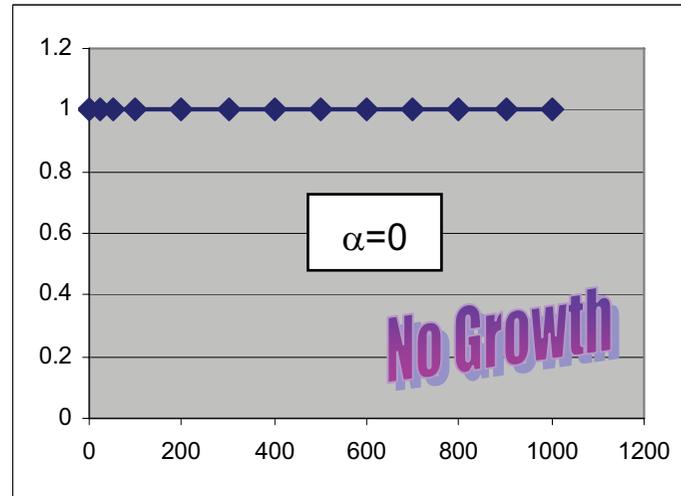
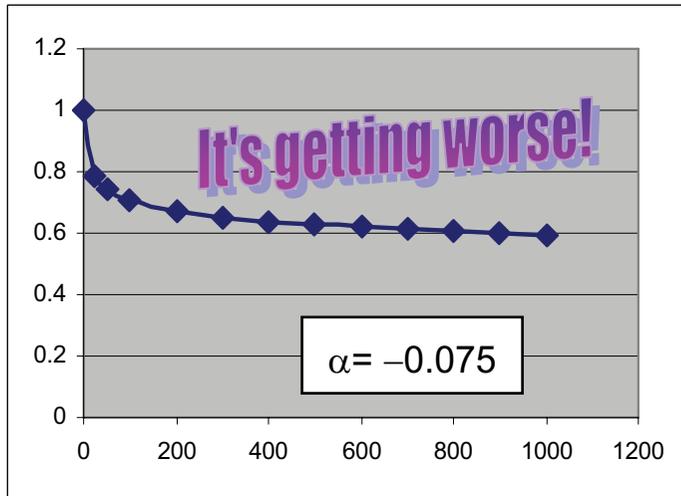
$$\text{Log}(\text{MTBF}) = \text{log}(\text{MTBF}_i) + \alpha \text{log}(T/T_i)$$

or

$$\text{MTBF} = \text{MTBF}_i (T/T_i)^\alpha$$



# MTBF/MTBF<sub>0</sub> vs. T/T<sub>0</sub> for Various values of $\alpha$



# Duane Model in Minitab-Data Entry

Cumulative Test Time (t hours)	Cumulative Failures N(t)
31	1
61	2
99	3
171	4
242	5
372	6
461	7
511	8
715	9
820	10
848	11



MINITAB - Untitled - [Worksheet 1 \*\*\*]

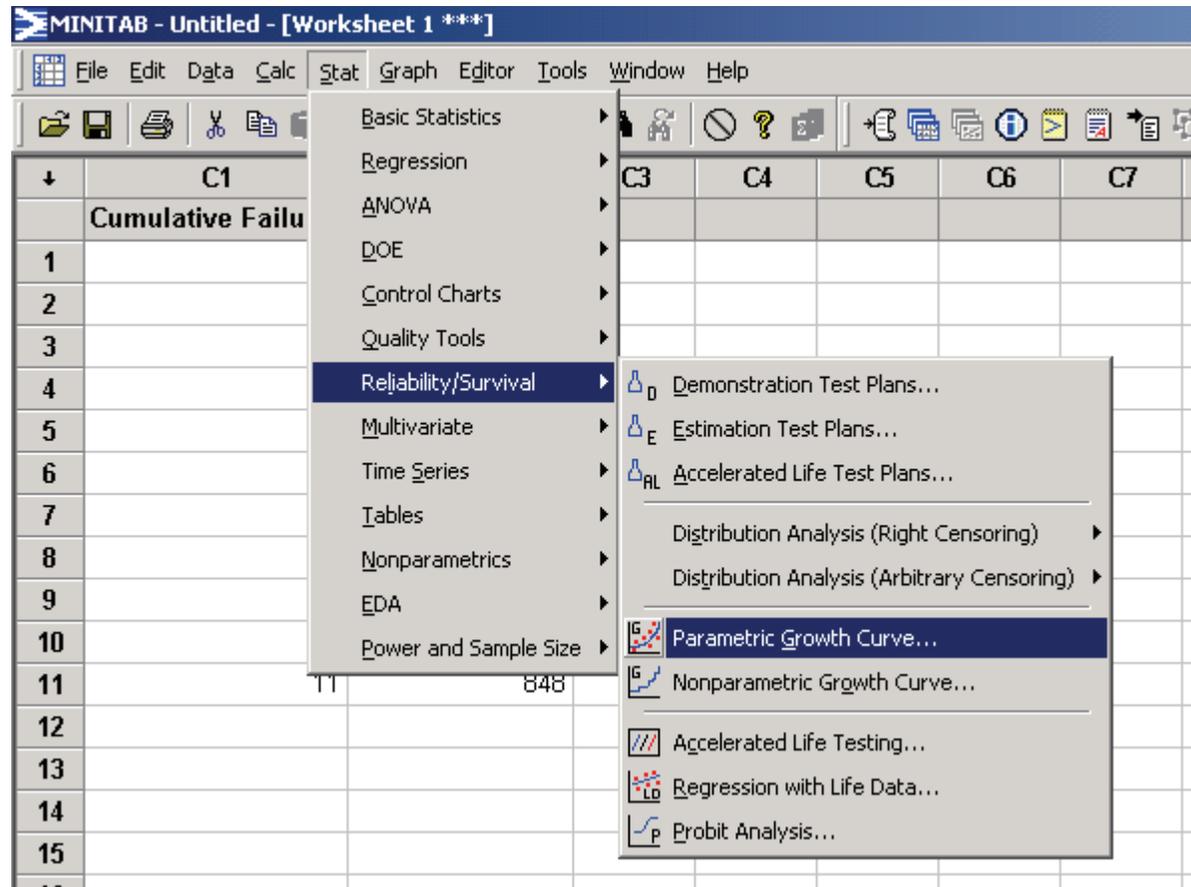
File Edit Data Calc Stat Graph Editor Io

↓ C1 C2

Cumulative Time to Failure

2	61	
3	99	
4	171	
5	242	
6	372	
7	461	
8	511	
9	715	
10	820	
11	848	
12		

# Duane Model in Minitab-Data Entry



# Minitab – Options

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## **Power-law process**

A nonhomogeneous Poisson process with an intensity function that represents the rate of failures or repairs. The power-law process can model a system that is improving, deteriorating, or remaining stable.

With the least squares estimation method, the power-law process model is also known as the Duane model.

# Minitab – Options

The image shows two overlapping dialog boxes from Minitab. The background dialog is titled "Parametric Growth Curve" and contains the following elements:

- Radio buttons for "Data are exact failure/retirement times" (selected) and "Data are interval failure/retirement times".
- Buttons for "Retirement...", "Estimate...", "Graphs...", "Results...", "Options...", and "Storage...".
- Fields for "Variables/Start variables:" (containing "'Cum. test time (t)')", "End variables:" (containing "'End Time'"), "Freq. columns: (optional)", "System ID: (optional)", and "By variable:".
- "Select" and "Help" buttons at the bottom.

The foreground dialog is titled "Parametric Growth Curve - Estimate" and contains the following elements:

- A dropdown menu for "Estimation method:" with "Least Squares" selected.
- Radio buttons for "Power-Law process" (selected), "Estimate shape parameter", "Set shape parameter:", and "Poisson process".
- A text field for "Confidence level:" containing "95.0".
- A dropdown menu for "Confidence intervals:" with "Two-sided" selected.
- "Help", "OK", and "Cancel" buttons at the bottom.

Red annotations highlight the "Estimate..." button in the main dialog, the "Least Squares" dropdown in the sub-dialog, and the "Power-Law process" radio button in the sub-dialog. A red arrow points from the "Estimate..." button to the sub-dialog.

# Minitab – Options

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Parametric Growth Curve - Graphs

Event plot  
 Display number of failures on event plot

Mean cumulative function (MCF) and Nelson-Aalen plot  
 Display confidence intervals on MCF plot

Duane plot

Total time on test (TTT) plot

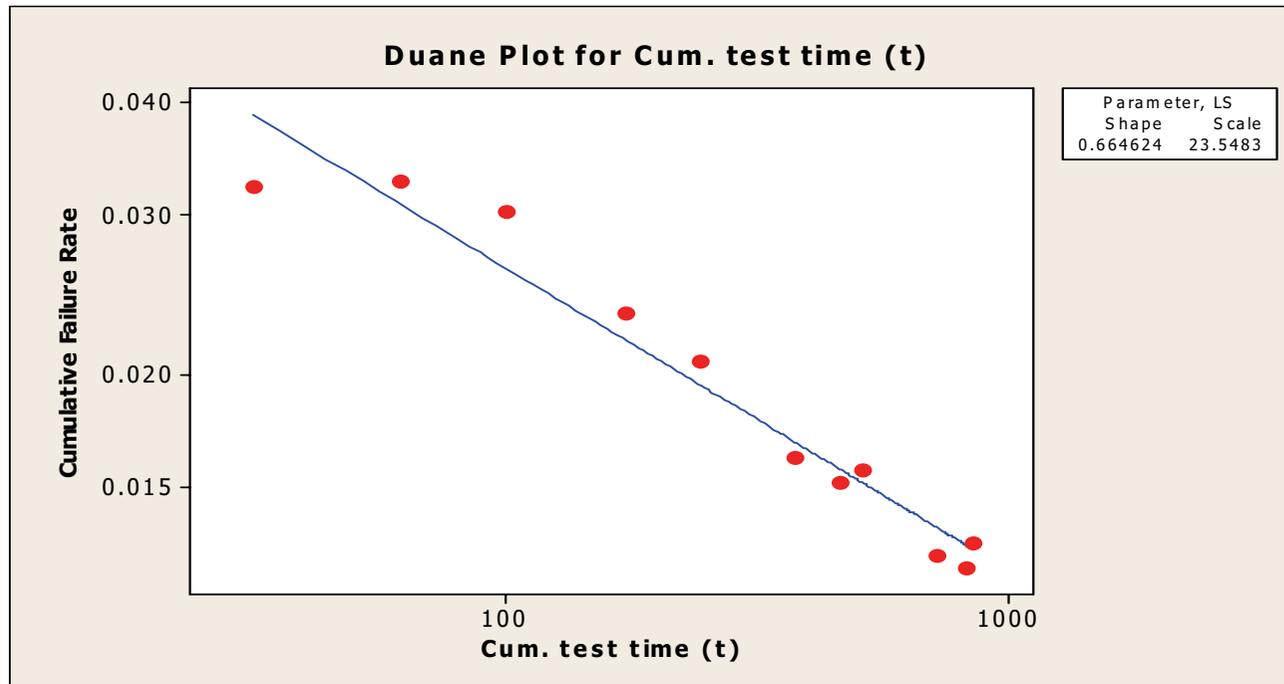
Display (except event plot):  
 an average plot over all systems  
 one plot per system

Show different variables/by levels or systems: Overlaid on the same graph

X axis display (except TTT plot):  
Minimum X scale:  Maximum X scale:   
X axis label:

Help OK Cancel

# Minitab – Duane Growth Plot



Since  $\beta$  (0.664624) is less than 1, or  $\alpha = 1 - \beta$ ,  $\alpha = 1 - 0.664 = 0.336$

Failure Rate has been decreasing  
i.e.,  
MTBF has been increasing

# Minitab – Options

The image displays two overlapping dialog boxes from Minitab. The top dialog is titled "Parametric Growth Curve" and contains the following elements:

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- Fields for "Freq. columns: (optional)", "System ID: (optional)", and "By variable:".
- Buttons for "Retirement...", "Estimate...", "Graphs...", "Results...", "Options...", and "Storage...".
- "Select" and "Help" buttons at the bottom.

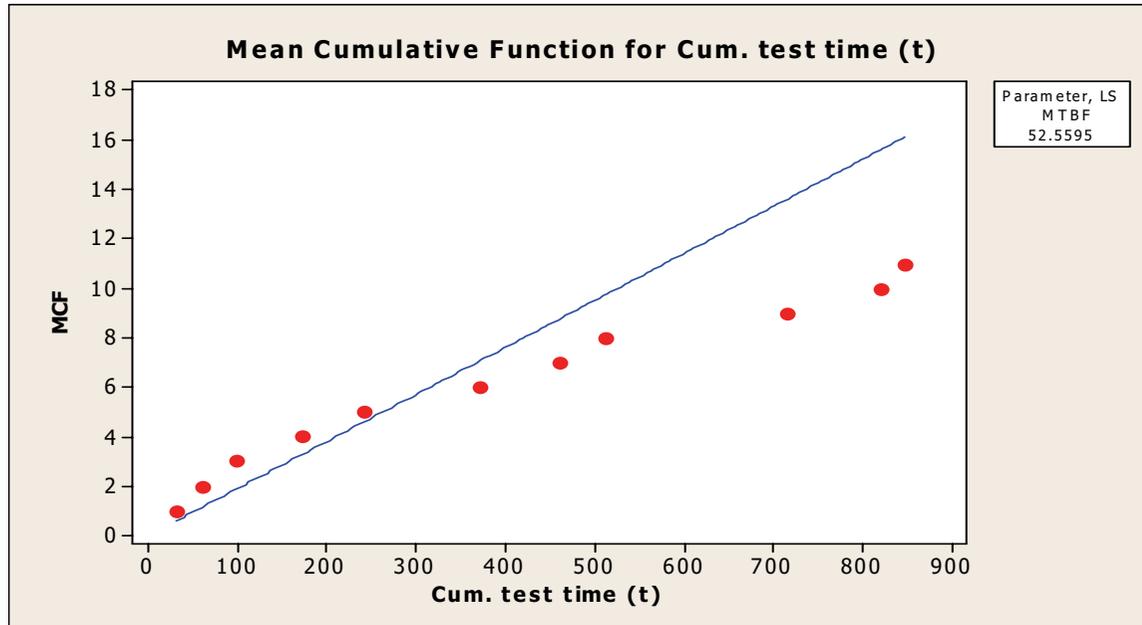
The bottom dialog is titled "Parametric Growth Curve - Estimate" and contains the following elements:

- "Estimation method:" dropdown menu with "Least Squares" selected (circled in red).
- Radio buttons for "Power-Law process" and "Poisson process" (circled in red).
- Sub-options for "Estimate shape parameter" (selected) and "Set shape parameter:".
- "Confidence level:" field set to "95.0".
- "Confidence intervals:" dropdown menu set to "Two-sided".
- "Help", "OK", and "Cancel" buttons at the bottom.

A red arrow points from the "Estimate..." button in the top dialog to the "Parametric Growth Curve - Estimate" dialog.

# Estimation of MTBF

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## Parameter Estimate

*Parameter*    *Estimate*

MTBF            52.5595

# Crow-AMSAA (CA) Model

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- Dr. Crow made significant contributions to Duane model while he was working at US Army-AMSAA (Army Material Systems Analysis Agency).
- Dr. Crow brought statistical interpretation of the Duane observation by proposing the Non-Homogenous Poisson process with a Weibull intensity function to model system failure times that follow the Duane Postulate.
- CA model is applied to repairable items only.
- CA model applies to times-to-failure data from a Test-Fix-Test RG process.

# CA Model Properties

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- Failure Times: Non-Homogenous Poisson Process
- Mean Value Function:  $E[N(t)] = \lambda t^\beta$
- Intensity Function:  $f(t) = \lambda \beta t^{\beta-1}$
- Cumulative MTBF =  $(\lambda t^{\beta-1})^{-1}$
- Instantaneous MTBF =  $(\lambda \beta t^{\beta-1})^{-1}$
- Model parameters  $\lambda$  and  $\beta$  are estimated using Maximum Likelihood Estimate:

$\beta$  = Shape parameter of the fitted line

$\lambda$  = Scale parameter of the fitted line

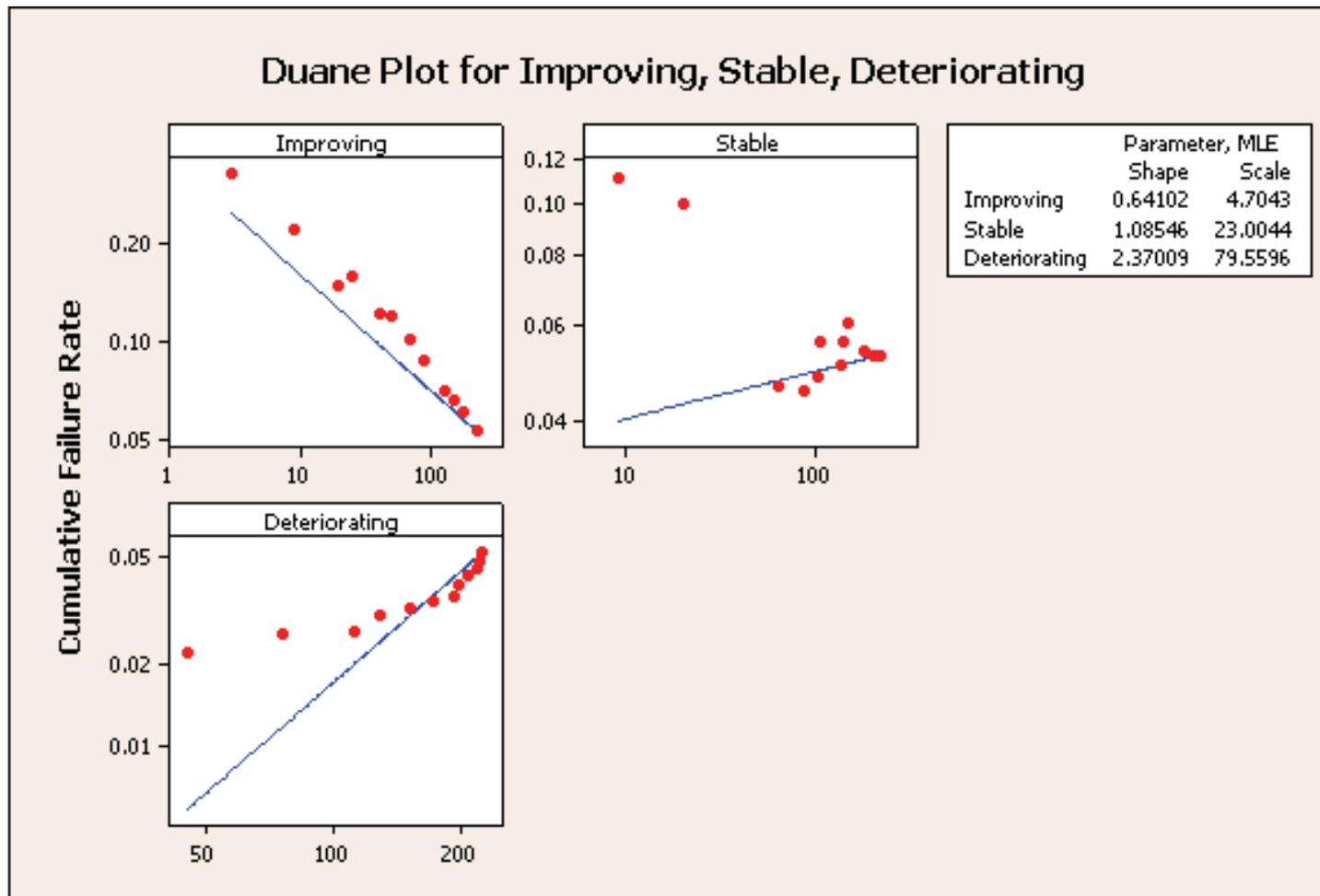
# Growth Rate

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The intensity function represents the rate of failures or repairs. The value of the shape ( $\beta$ ) depends on whether your system is improving, deteriorating, or remaining stable based on the ( $\beta$ ) as follows:

- If  $0 < \beta < 1$ , the failure/repair rate is decreasing. Thus, your system is improving over time.
- If  $\beta = 1$ , the failure/repair rate is constant. Thus, your system is remaining stable over time.
- If  $\beta > 1$ , the failure/repair rate is increasing. Thus, your system is deteriorating over time.

# Growth Rate



# The Empirical Model

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Assumed relationship between  $N(t)$  and  $t$ :

$$N(t) = \lambda t^\beta \dots (1)$$

Taking natural logarithm we obtain:

$$\ln N(t) = \ln(\lambda) + \beta \ln(t)$$

Thus, the equation (1) is a straight line when plotted on log-log graph paper.

# Crow-AMSAA Model: Procedure

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- Accumulates failure data during cycles of TFT (or field failure analysis) by continually plotting cumulative number of failures against cumulative test time on log-log scale
- Determines the straight line of best fit through the scatter plot (equation 1)
- Determines the slope of the fitted line. The slope indicates whether the failure rate has been decreasing (i.e., MTBF increasing) during the improvement cycles.
- Estimates the instantaneous MTBF achieved at any point during the improvement cycles.

# Example: Reliability Growth

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The following failure data were generated through the TFT cycles for an electro-mechanical actuator during its development phase.

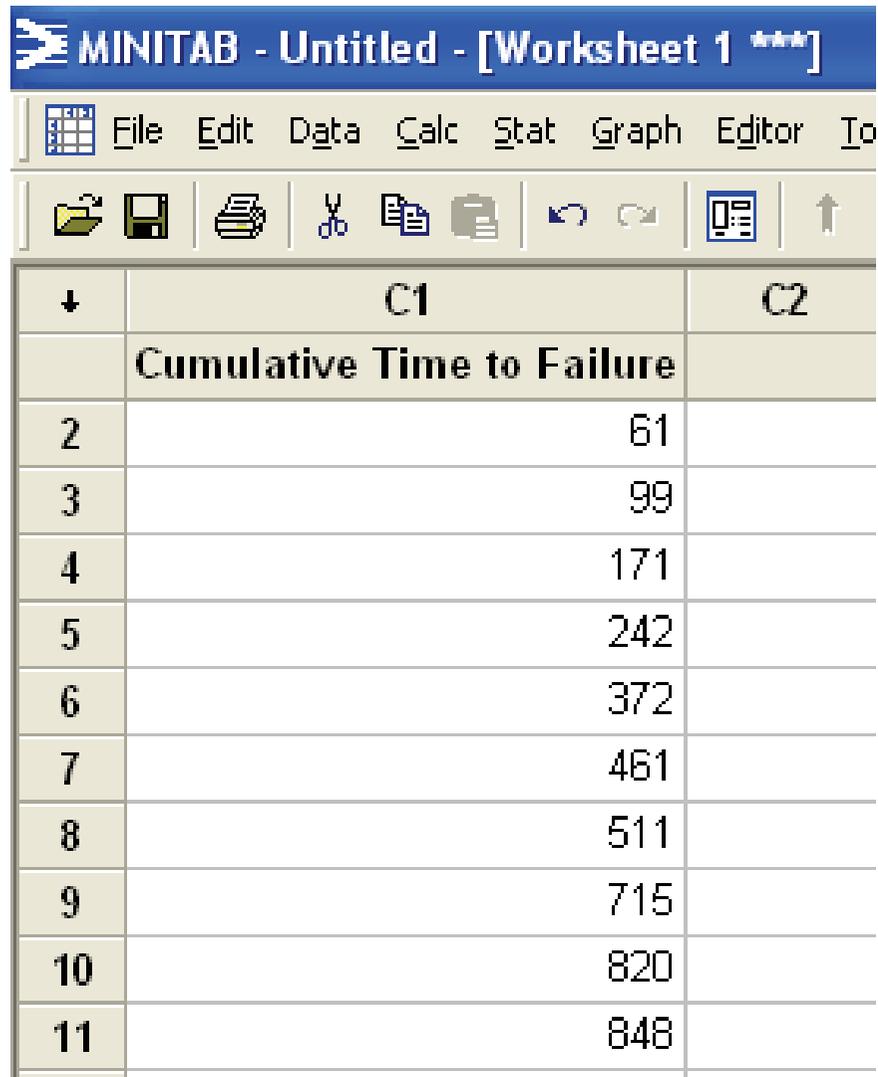
Based on the data:

1. Asses Reliability Growth?
2. Estimate MTBF?

<b>Cumulative Test Time (t hours)</b>	<b>Cumulative Failures N(t)</b>
31	1
61	2
99	3
171	4
242	5
372	6
461	7
511	8
715	9
820	10
848	11

# Minitab – Data Entry

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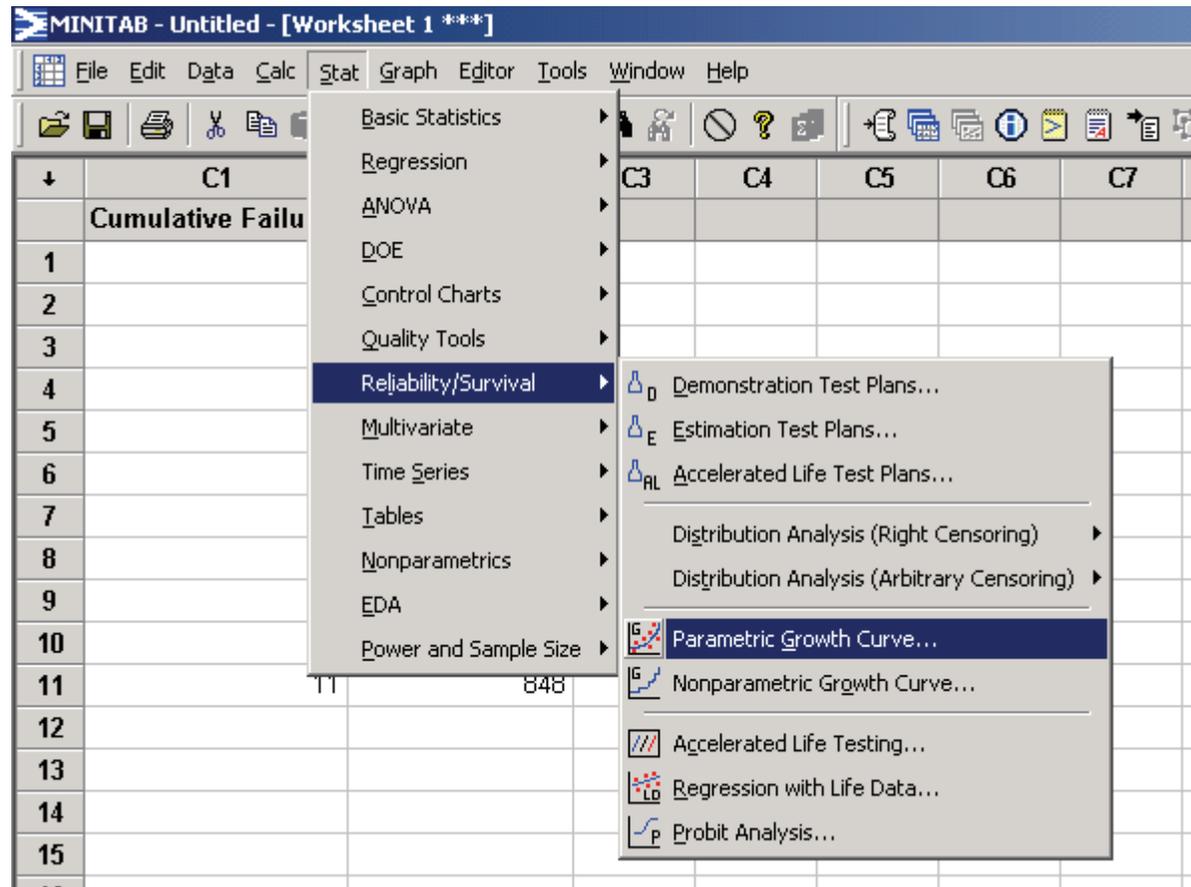


The screenshot displays the Minitab software interface. The title bar reads "MINITAB - Untitled - [Worksheet 1 \*\*\*]". The menu bar includes "File", "Edit", "Data", "Calc", "Stat", "Graph", "Editor", and "Tools". The toolbar contains icons for opening a file, saving, printing, cutting, pasting, undo, redo, and a cursor icon. The main data table is as follows:

+	C1	C2
	<b>Cumulative Time to Failure</b>	
2	61	
3	99	
4	171	
5	242	
6	372	
7	461	
8	511	
9	715	
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11	848	

# Crow-AMSAA Model in Minitab – Data Entry

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# Minitab – Options

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## **Power-law process**

A non-homogeneous Poisson process with an intensity function that represents the rate of failures or repairs. The power-law process can model a system that is improving, deteriorating, or remaining stable.

With the default (maximum likelihood) estimation method, the power-law model is also known as the Crow-AMSAA model.

# Minitab – Options

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- Fields for "Freq. columns: (optional)" and "System ID: (optional)".
- A checkbox for "By variable:".
- Buttons for "Retirement...", "Estimate...", "Graphs...", "Results...", "Options...", and "Storage...".
- "Select" and "Help" buttons at the bottom.

The foreground dialog is titled "Parametric Growth Curve - Estimate" and contains the following elements:

- A dropdown menu for "Estimation method:" with "Maximum Likelihood" selected.
- Radio buttons for "Power-Law process" (selected), "Estimate shape parameter" (selected), "Set shape parameter:", and "Poisson process".
- A text field for "Confidence level:" with the value "95.0".
- A dropdown menu for "Confidence intervals:" with "Two-sided" selected.
- "Help", "OK", and "Cancel" buttons at the bottom.

Red annotations highlight the "Estimate..." button in the main dialog, the "Maximum Likelihood" method in the sub-dialog, and the "Estimate shape parameter" option in the sub-dialog. A red arrow points from the "Estimate..." button to the sub-dialog.

# Minitab – Options

Parametric Growth Curve - Graphs

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one plot per system

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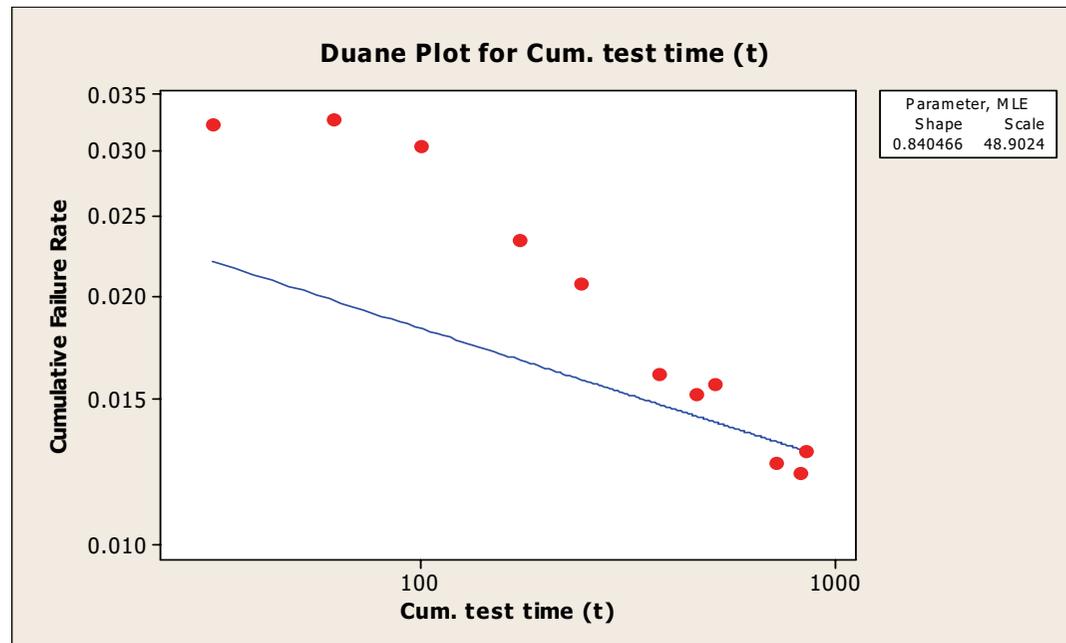
X axis display (except TTT plot):

Minimum X scale:  Maximum X scale:

X axis label:

Help OK Cancel

# Minitab – Data Analysis



Since  $\beta$  (0.840) is less than 1, This implies that:

Failure Rate has been decreasing

i.e.,

MTBF has been increasing

# Minitab – Options

The image shows two overlapping dialog boxes from Minitab. The top dialog is titled "Parametric Growth Curve" and contains the following elements:

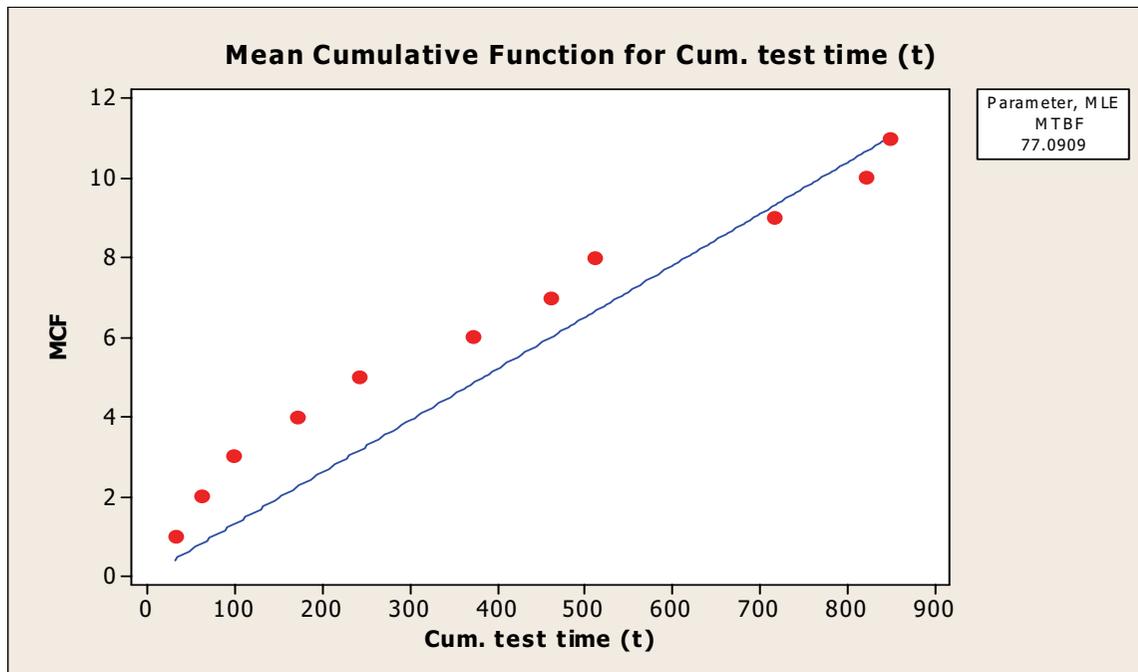
- Variable list: C1 (Cum. test t), C2 (Cum. Failure)
- Radio buttons:  Data are exact failure/retirement times;  Data are interval failure/retirement times
- Buttons: Retirement..., Estimate..., Graphs..., Results..., Options..., Storage...
- Fields: Variables/Start variables: 'Cum. test time (t)'; End variables: 'End Time'; Freq. columns: (optional); System ID: (optional); By variable: (checkbox and field)
- Buttons: Select, Help

The bottom dialog is titled "Parametric Growth Curve - Estimate" and contains the following elements:

- Estimation method: Maximum Likelihood (circled in red)
- Radio buttons:  Power-Law process;  Poisson process (circled in red)
- Options:  Estimate shape parameter;  Set shape parameter: (field)
- Confidence level: 95.0
- Confidence intervals: Two-sided
- Buttons: Help, OK, Cancel

A red arrow points from the "Estimate..." button in the top dialog to the "Estimate" sub-dialog.

# MTBF Estimation



## Parameter Estimate

	Standard	95% Normal CI	
Parameter Estimate	Error	Lower	Upper
MTBF	77.0909	23.244	139.203