RG Models Used in TFT

- **Crow-AMSAA**
- **Duane**
- Gompertz and Modified Gompertz
- Lloyd Lipow
- Logistic

Duane Model

- Also called as “Duane Postulate”.
- Developed based on an observation by J.T. Duane.
- Duane observed that the cumulative failure rate vs cumulative test time was linear on a log-log paper when he plotted test data from generators, hydro-mechanical devices and aircraft engines.
- It is a **deterministic model**.
- Used for Test-Fix-Test data.

Duane Growth Model

$$MTBF_C = MTBF_0\left(\frac{T}{T_0}\right)^{\alpha}$$

- $MTBF_C$ is cumulative=total failures/total time.
- $MTBF_0$ is at starting point.
- T_0 is starting time and α is the growth rate.
- Can be used to estimate required test time to reach goals or if time is fixed, determine how many systems must be tested .
- With good effort α~0.3-0.4 for complex systems.
- Instantaneous $MTBF_i$=MTBF/(1- α).

Meaning of Alpha Values

- The slope, alpha, gives an indication of the rate of MTBF growth.
- Shows effectiveness of the company's reliability program.
- Duane observed that alpha ranged between 0.2 and 0.4 and that range of values was a very good correlation with the intensity of the effort on the reliability growth program.
- The higher the alpha value, the bigger the effects of the improvements (reliability growth)in the reliability of the item.
- When alpha <0 or alpha = 0, there is NO Growth.

Duane Growth Model – Example

$$MTBF_C = MTBF_0\left(\frac{T}{T_0}\right)^{\alpha}$$

- Let's see how this function looks over time with a range of growth rates.
- Let's look at the following cases and plot
- $MTBF_C / MTBF_0$ vs. T/T_0

  α=-0.075

  α=0.0

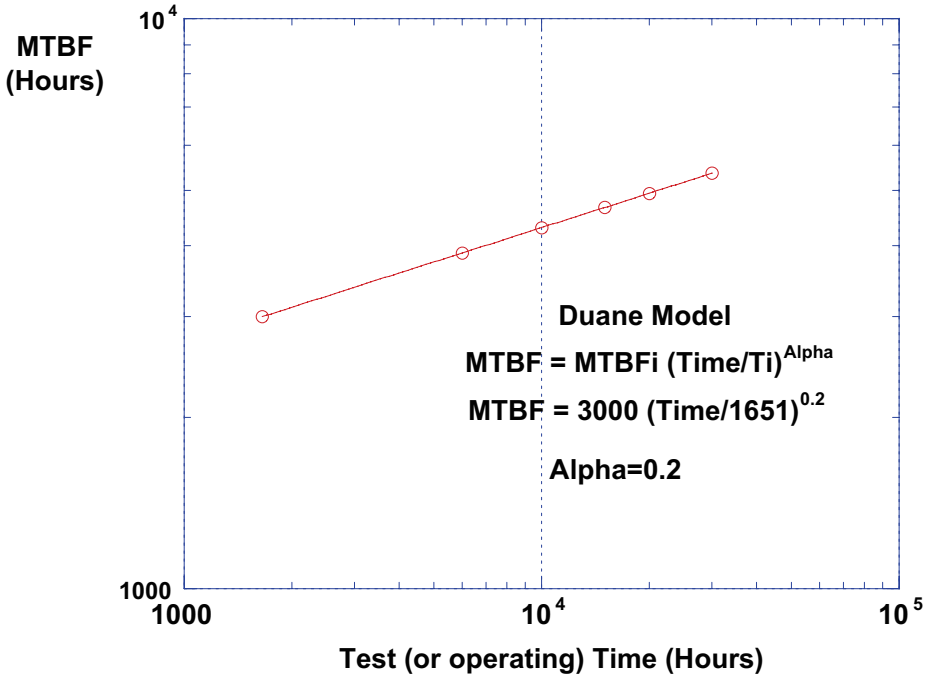
  α=0.3

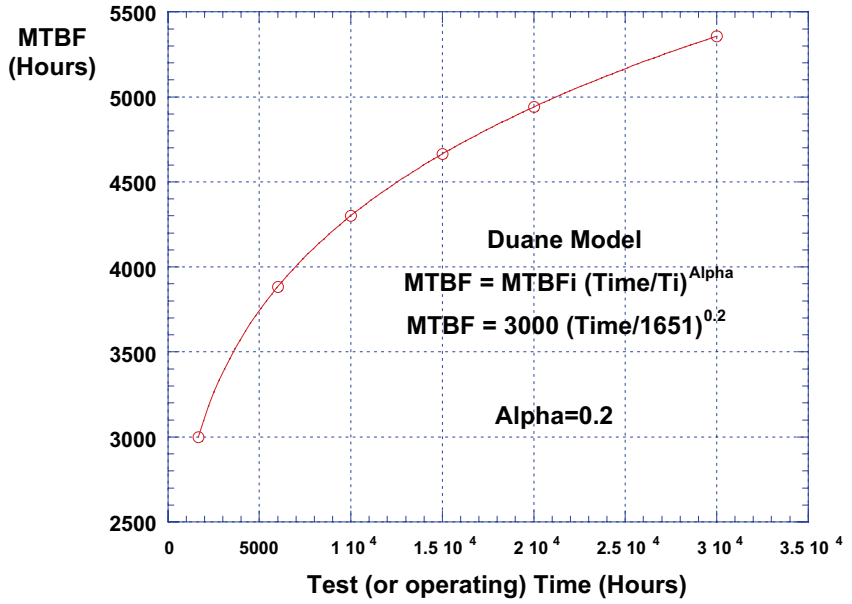
  α=1.0

Duane Reliability Growth Model

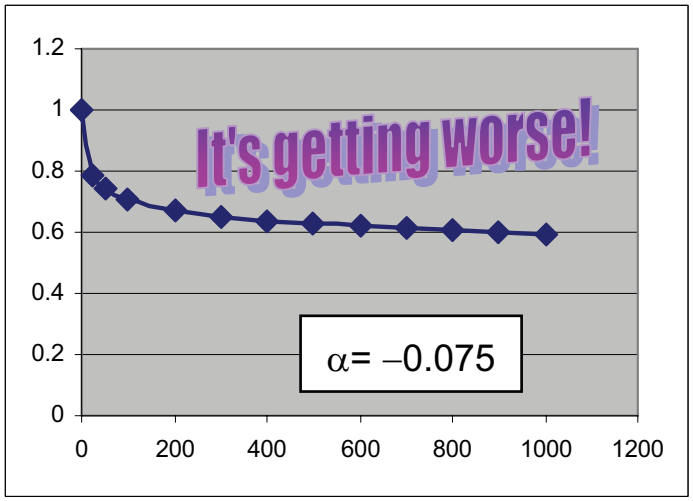
- Duane obtained a straight line on a log - log plot of MTBF versus operating time

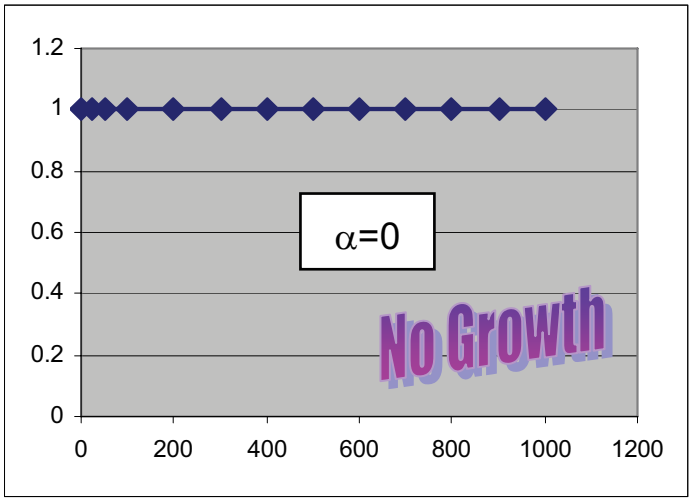
$\log(MTBF)=\log(MTBFi)+\alpha \log(T/Ti)$ or $MTBF=MTBFi\ (T/Ti)^{\alpha}$

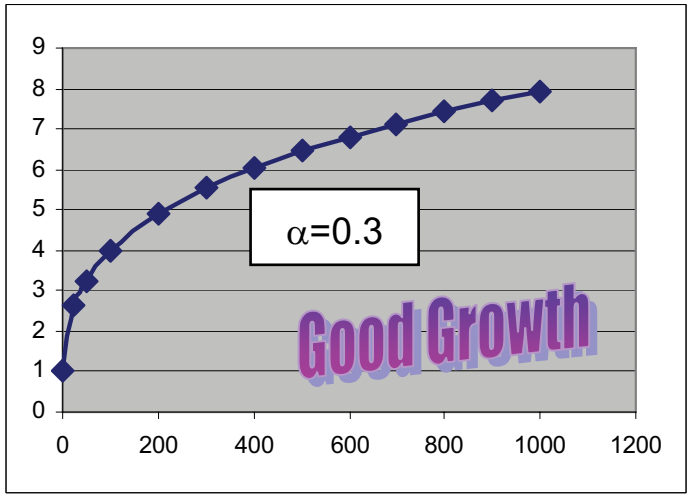


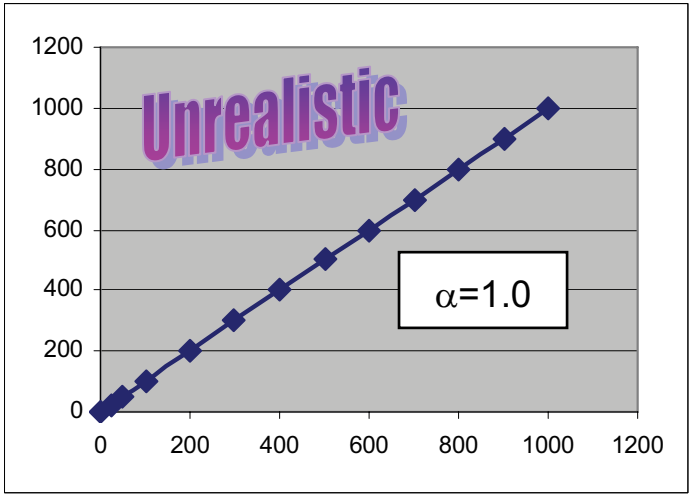


$MTBF/MTBF_0$ vs. T/T_0 for Various values of α



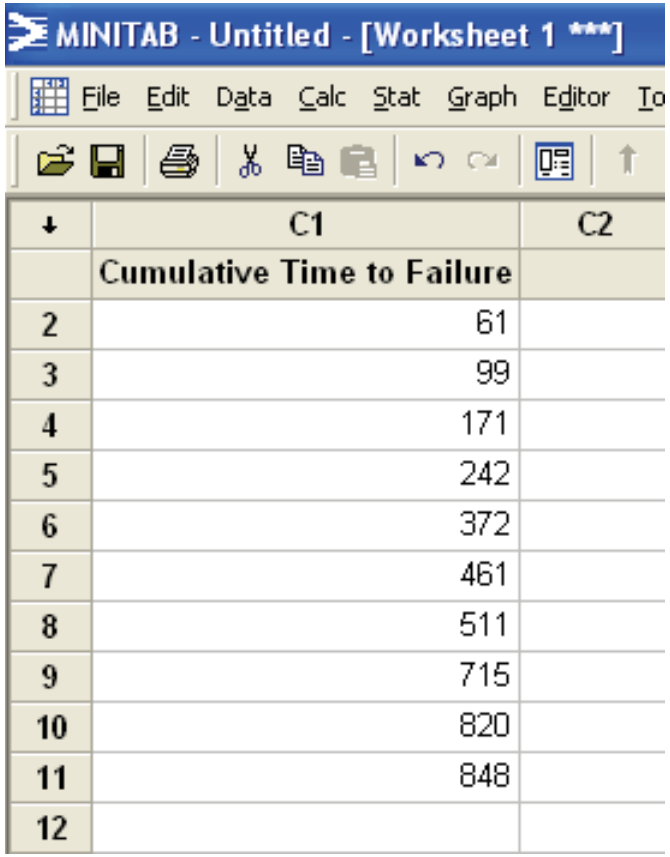






Duane Model in Minitab-Data Entry

Cumulative Test Time (t hours)	Cumulative Failures N(t)
31	1
61	2
99	3
171	4
242	5
372	6
461	7
511	8
715	9
820	10
848	11



↓	C1	C2
	Cumulative Time to Failure	
2	61	
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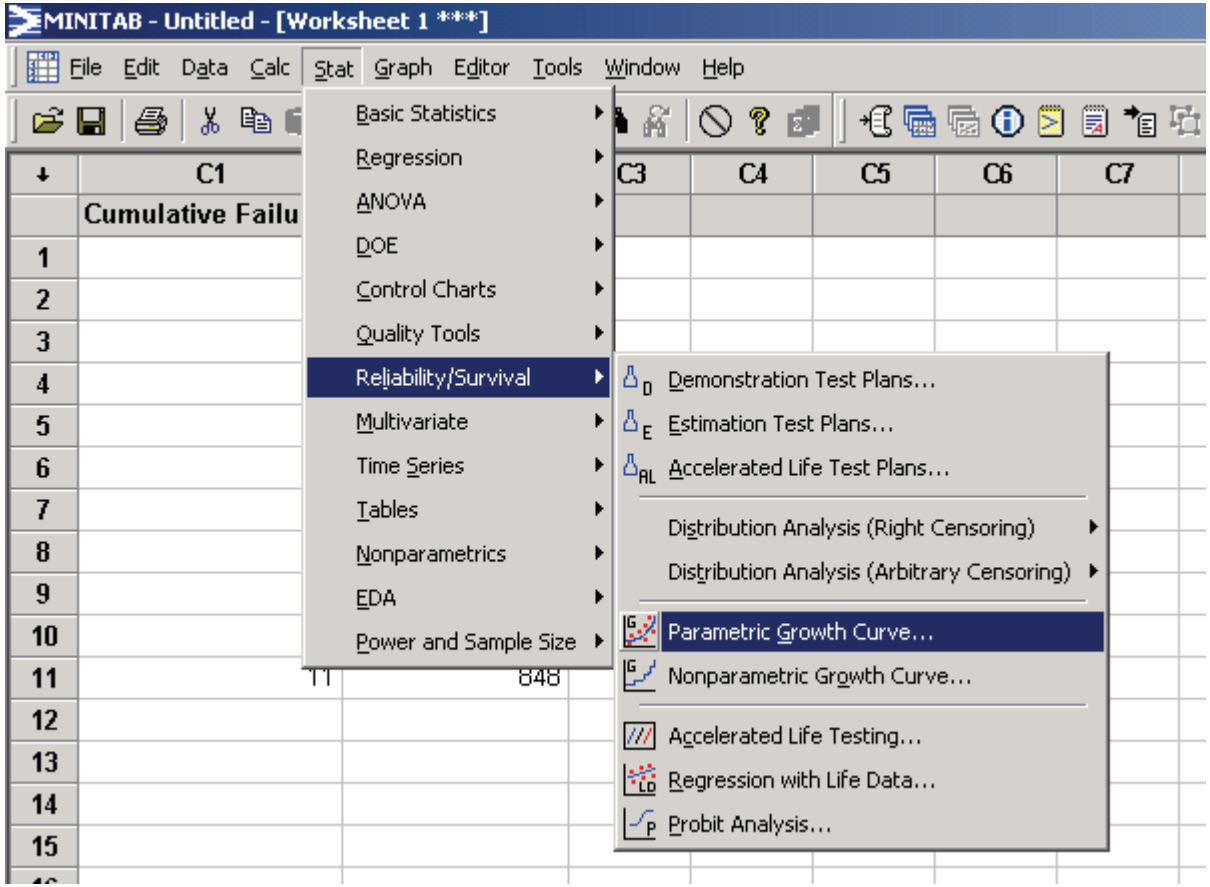
11	848	
12		

Duane Model in Minitab-Data Entry



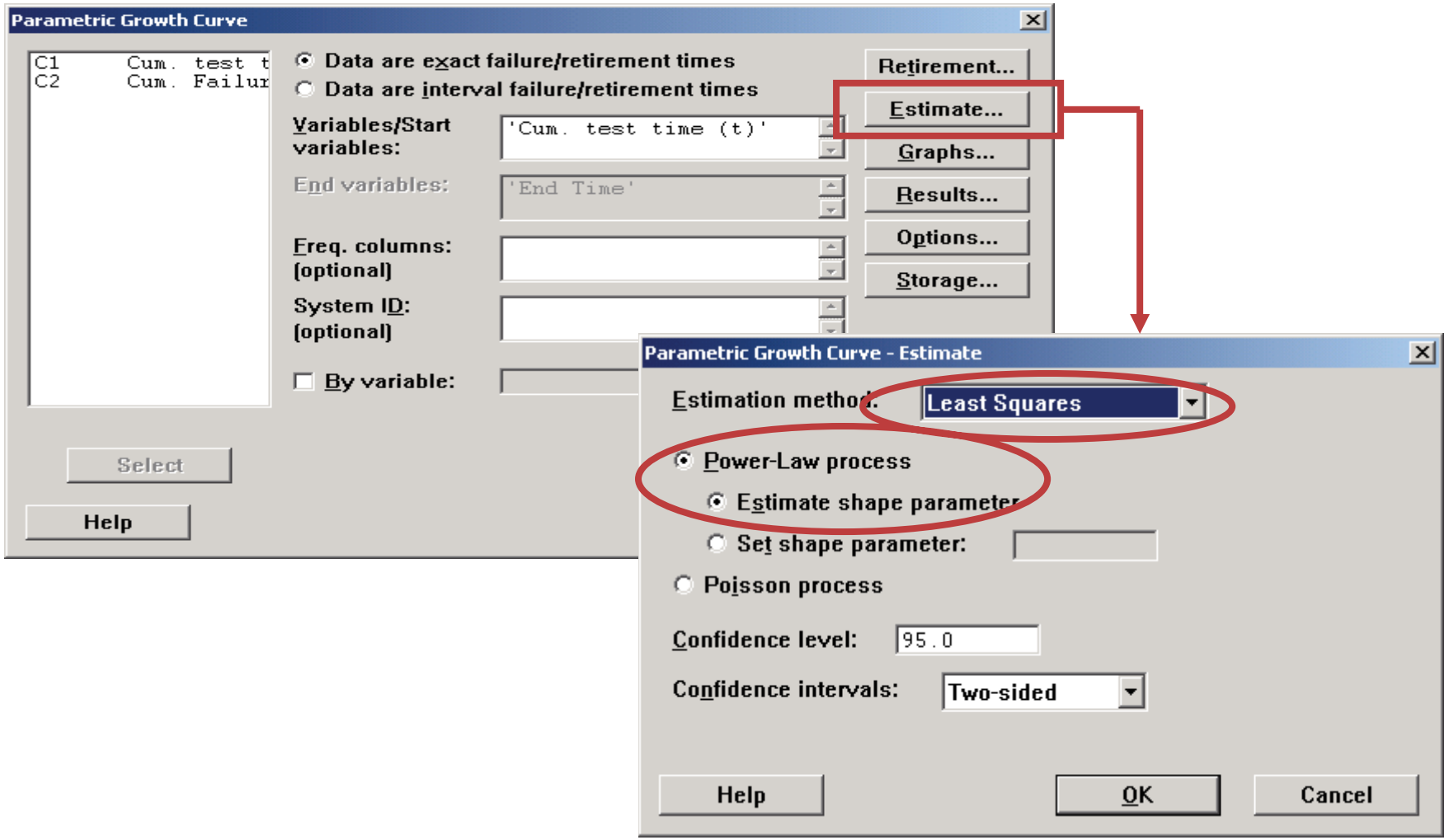
Minitab – Options

Power-law process

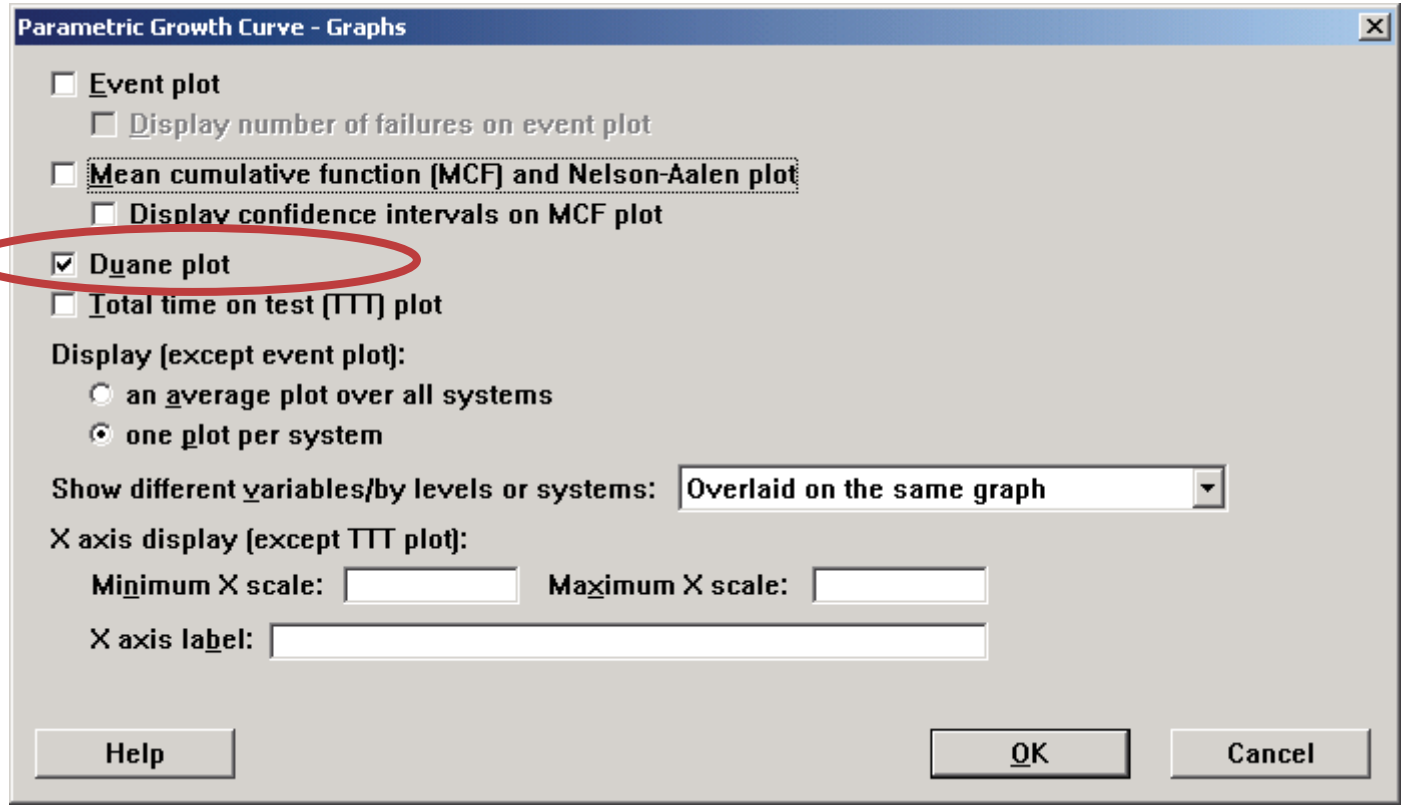
A nonhomogeneous Poisson process with an intensity function that represents the rate of failures or repairs. The power-law process can model a system that is improving, deteriorating, or remaining stable.

With the least squares estimation method, the power-law process model is also known as the Duane model.

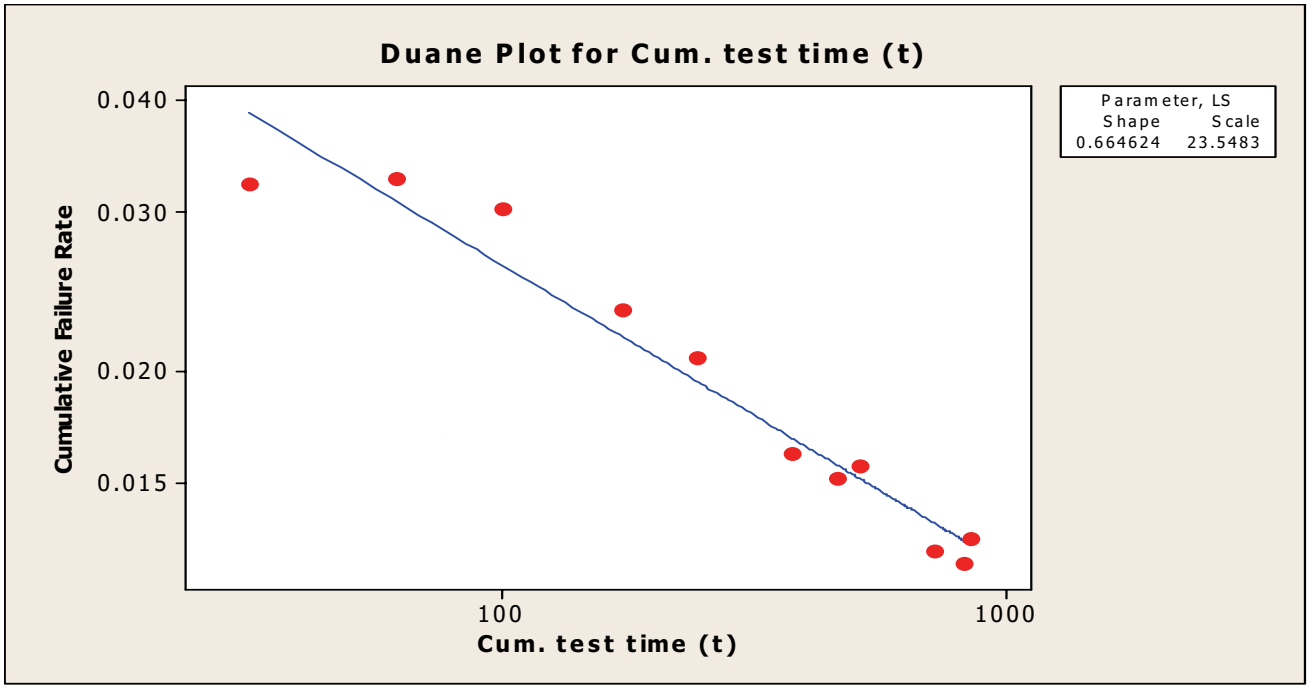
Minitab – Options



Minitab – Options



Minitab – Duane Growth Plot



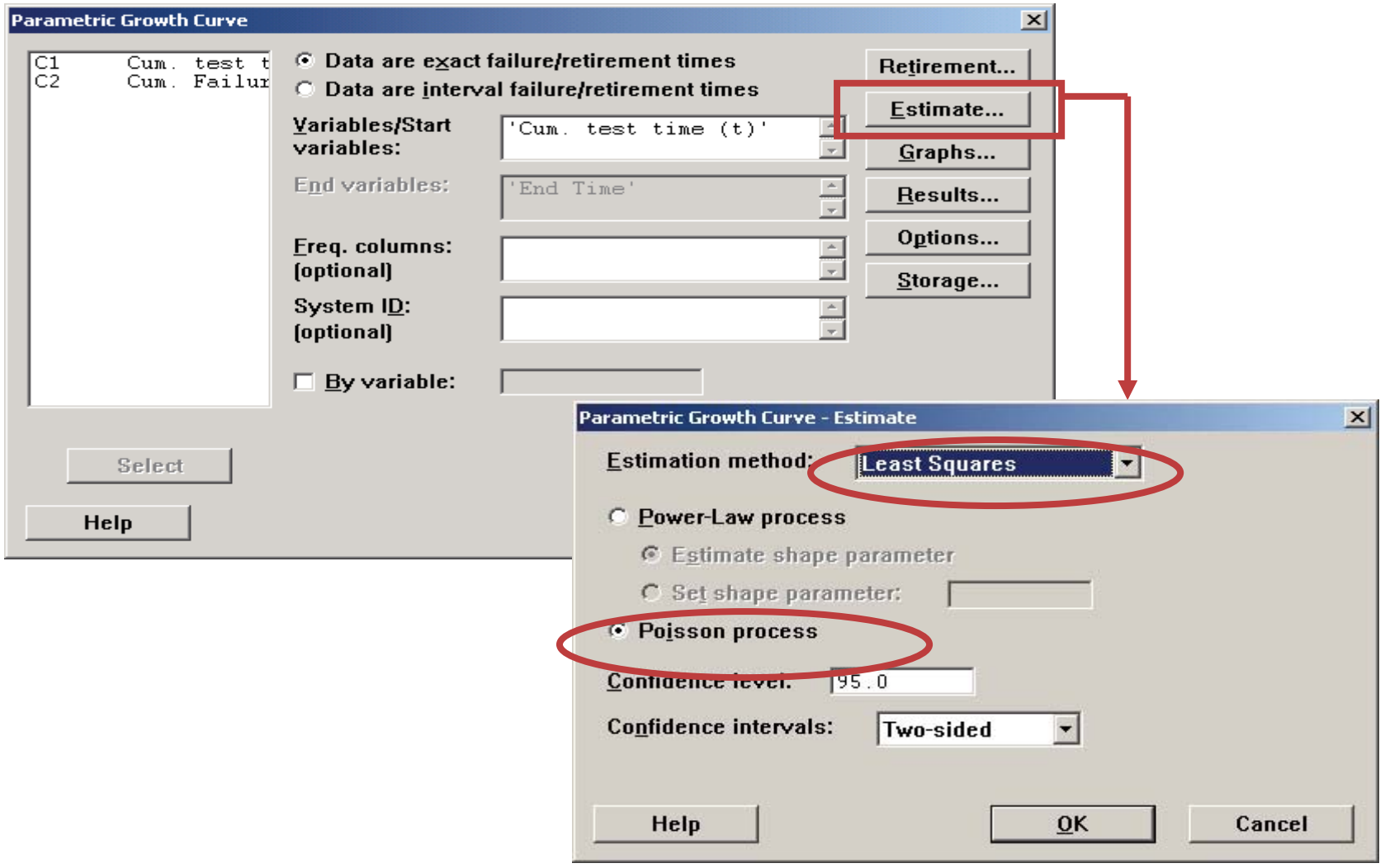
Since β (0.664624) is less than 1, or $\alpha = 1 - \beta$, α = 1-0.664 = 0.336

Failure Rate has been decreasing

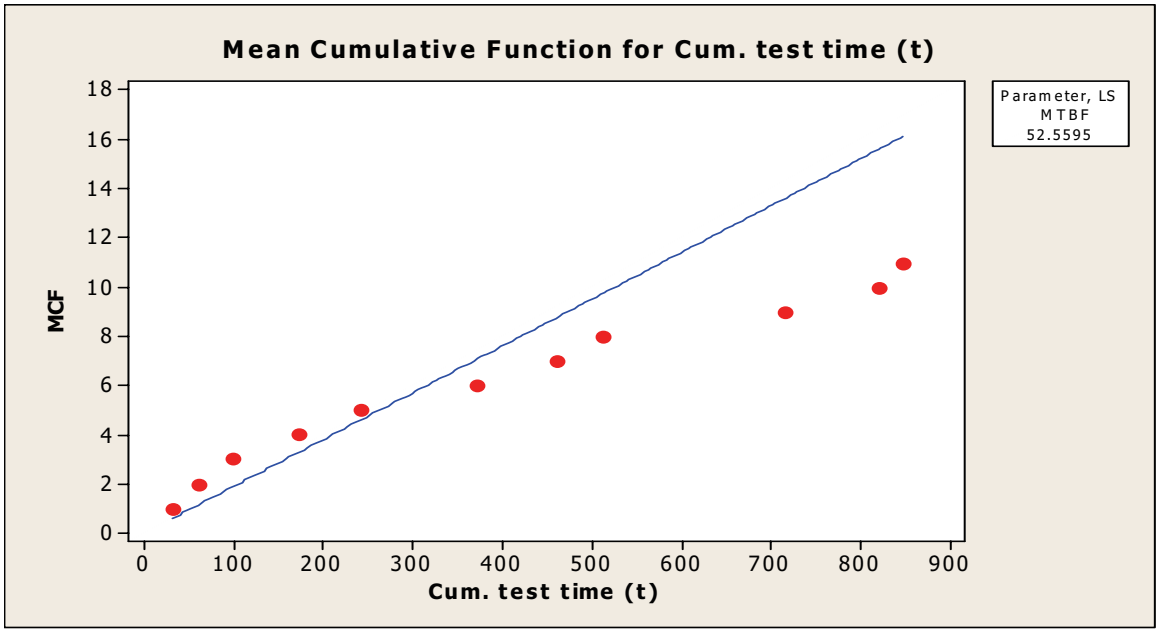
i.e.,

MTBF has been increasing

Minitab – Options



Estimation of MTBF



Parameter Estimate

Parameter	*Estimate*
MTBF	52.5595

Crow-AMSAA (CA) Model

- Dr. Crow made significant contributions to Duane model while he was working at US Army-AMSAA (Army Material Systems Analysis Agency).
- Dr. Crow brought statistical interpretation of the Duane observation by proposing the Non-Homogenous Poisson process with a Weibull intensity function to model system failure times that follow the Duane Postulate.
- CA model is applied to repairable items only.
- CA model applies to times-to-failure data from a Test-Fix-Test RG process.

CA Model Properties

- Failure Times: Non-Homogenous Poisson Process
- Mean Value Function: $E[N(t)] = \lambda t^{\beta}$
- Intensity Function: $f(t) = \lambda\beta t^{\beta-1}$
- Cumulative MTBF = $(\lambda t^{\beta-1})^{-1}$
- Instantaneous MTBF = $\left(\lambda\beta t^{\beta-1}\right)^{-1}$
- Model parameters λ and β are estimated using Maximum Likelihood Estimate:

  β = Shape parameter of the fitted line

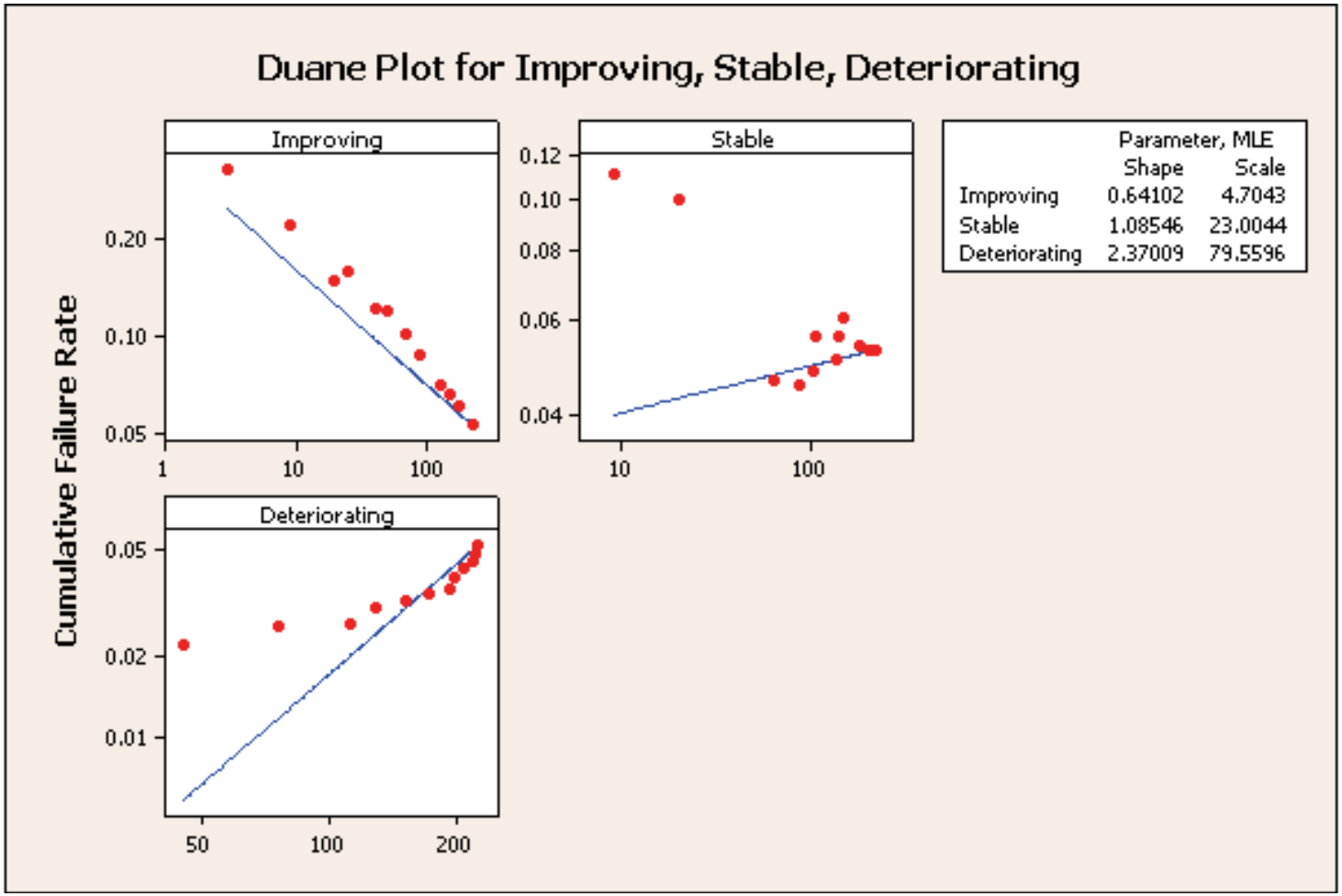
  λ = Scale parameter of the fitted line

Growth Rate

The intensity function represents the rate of failures or repairs. The value of the shape (β) depends on whether your system is improving, deteriorating, or remaining stable based on the (β) as follows:

- If $0 < \beta < 1$, the failure/repair rate is decreasing. Thus, your system is improving over time.
- If $\beta = 1$, the failure/repair rate is constant. Thus, your system is remaining stable over time.
- If $\beta > 1$, the failure/repair rate is increasing. Thus, your system is deteriorating over time.

Growth Rate


Duane Plot for Improving, Stable, Deteriorating
Improving
Stable
Deteriorating
Cumulative Failure Rate
0.20
0.10
0.05
1
10
100
0.12
0.10
0.08
0.06
0.04
10
100
0.05
0.02
0.01
50
100
200
Parameter, MLE
Shape
Scale
Improving 0.64102 4.7043
Stable 1.08546 23.0044
Deteriorating 2.37009 79.5596

The Empirical Model

Assumed relationship between N(t) and t:

$$N(t) = \lambda t^{\beta} \ldots (1)$$

Taking natural logarithm we obtain:

$$\ln N(t) = \ln(\lambda) + \beta \ln(t)$$

Thus, the equation (1) is a straight line when plotted on log-log graph paper.

Crow-AMSAA Model: Procedure

- Accumulates failure data during cycles of TFT (or field failure analysis) by continually plotting cumulative number of failures against cumulative test time on log-log scale
- Determines the straight line of best fit through the scatter plot (equation 1)
- Determines the slope of the fitted line. The slope indicates whether the failure rate has been decreasing (i.e., MTBF increasing) during the improvement cycles.
- Estimates the instantaneous MTBF achieved at any point during the improvement cycles.

Example: Reliability Growth

The following failure data were generated through the TFT cycles for an electro-mechanical actuator during its development phase.

Based on the data:

1. Asses Reliability Growth?
2. Estimate MTBF?

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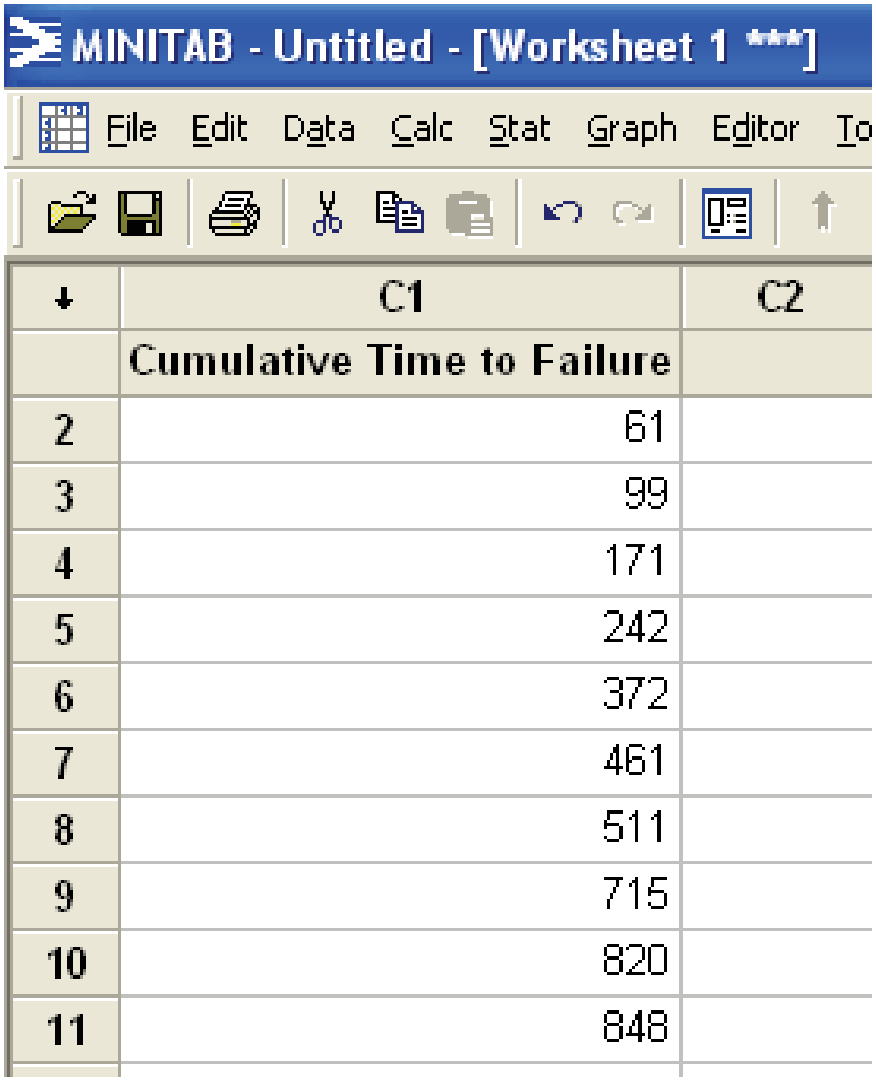
Minitab – Data Entry



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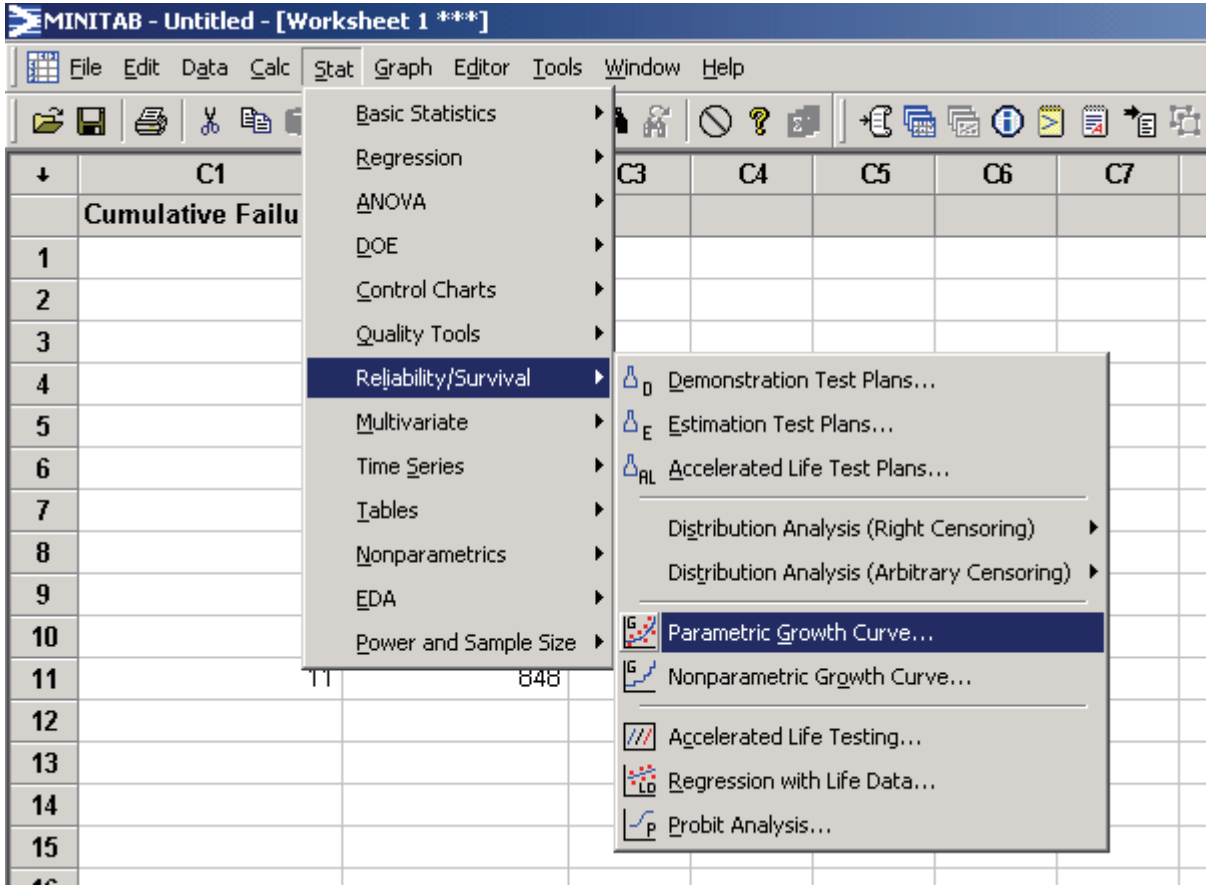

Minitab – Options

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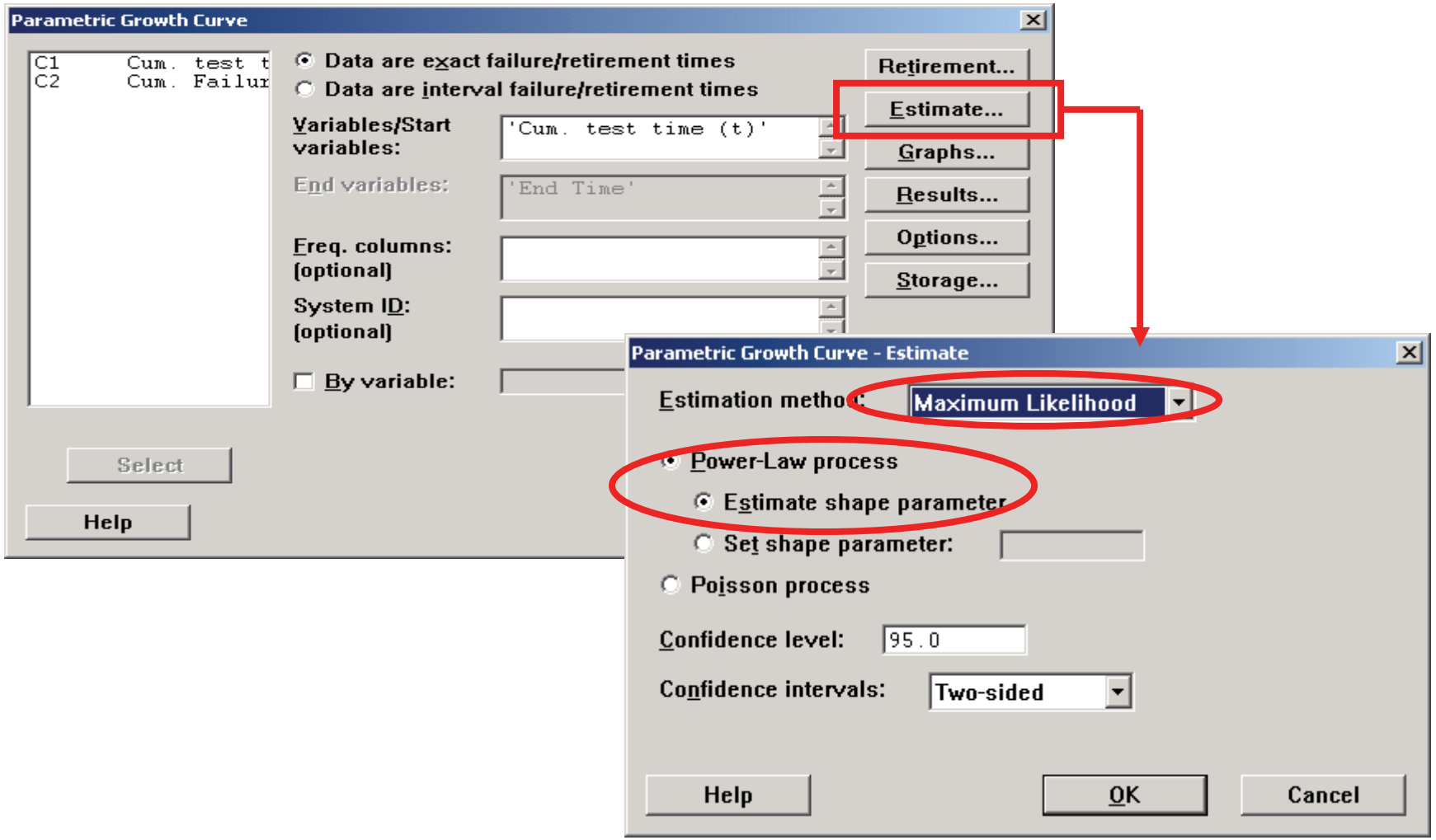
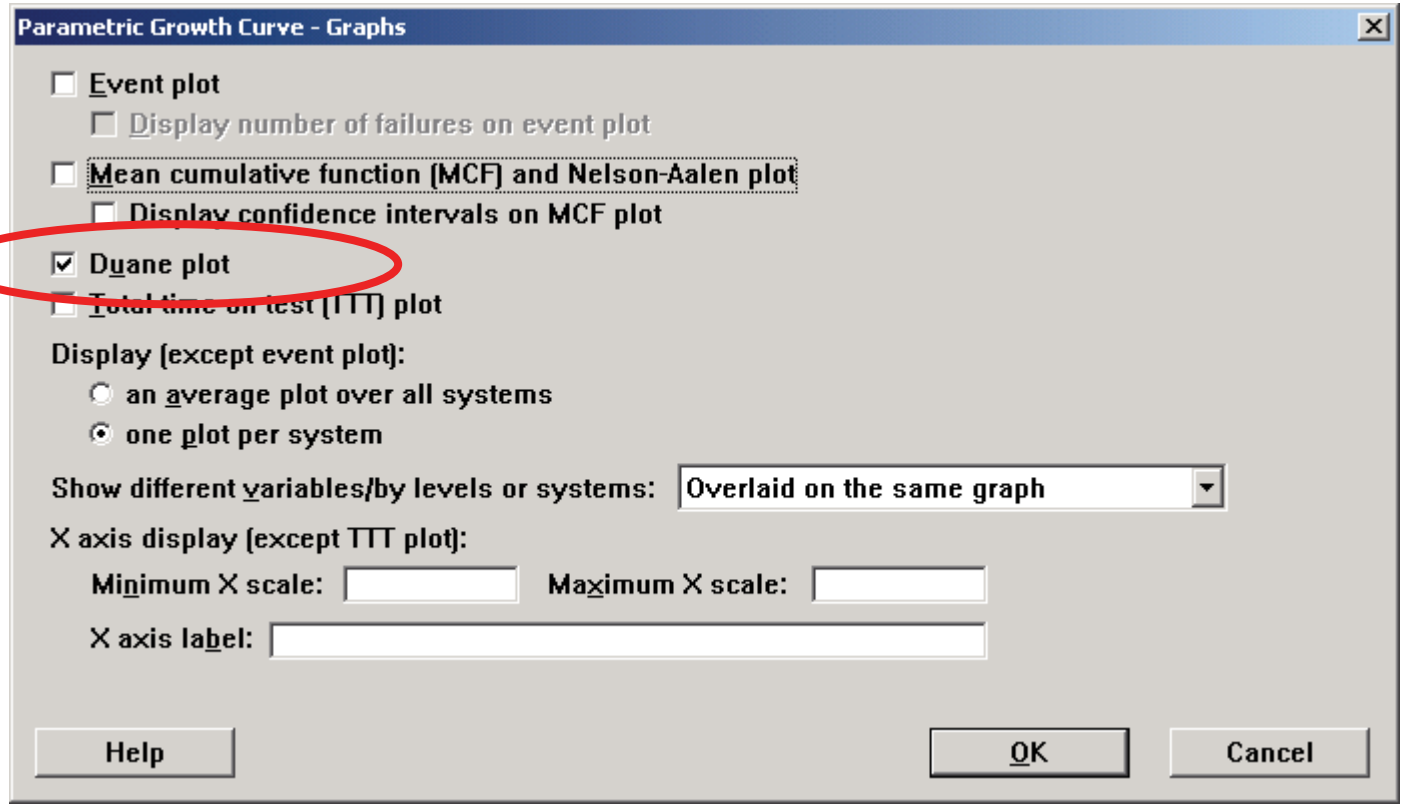
With the default (maximum likelihood) estimation method, the power-law model is also known as the Crow-AMSAA model.

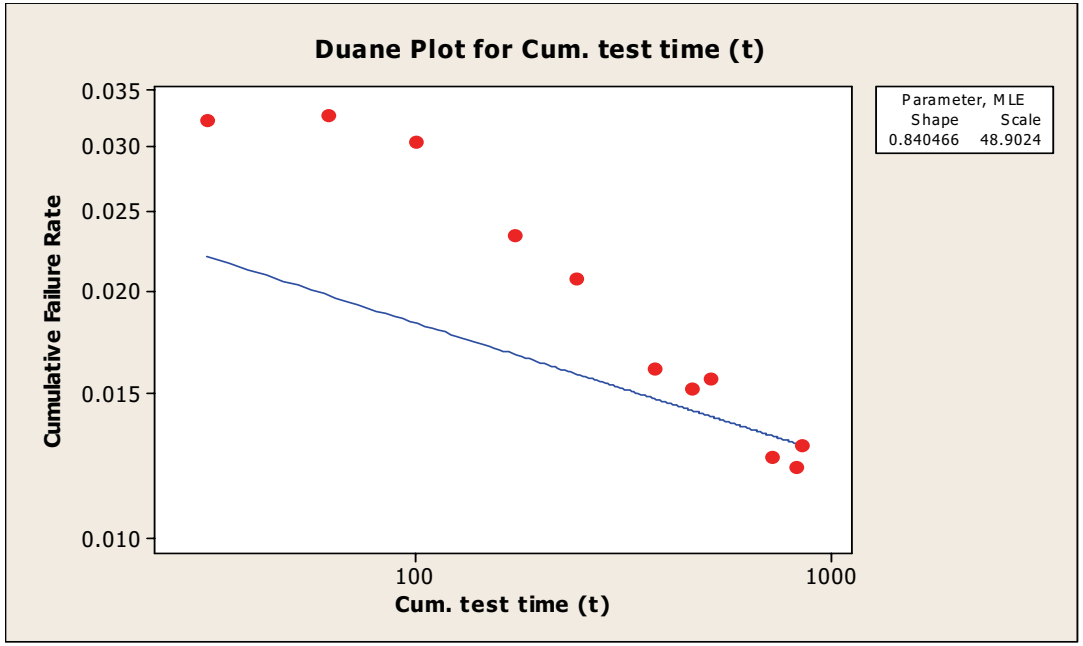
Minitab – Options



Minitab – Options



Minitab – Data Analysis



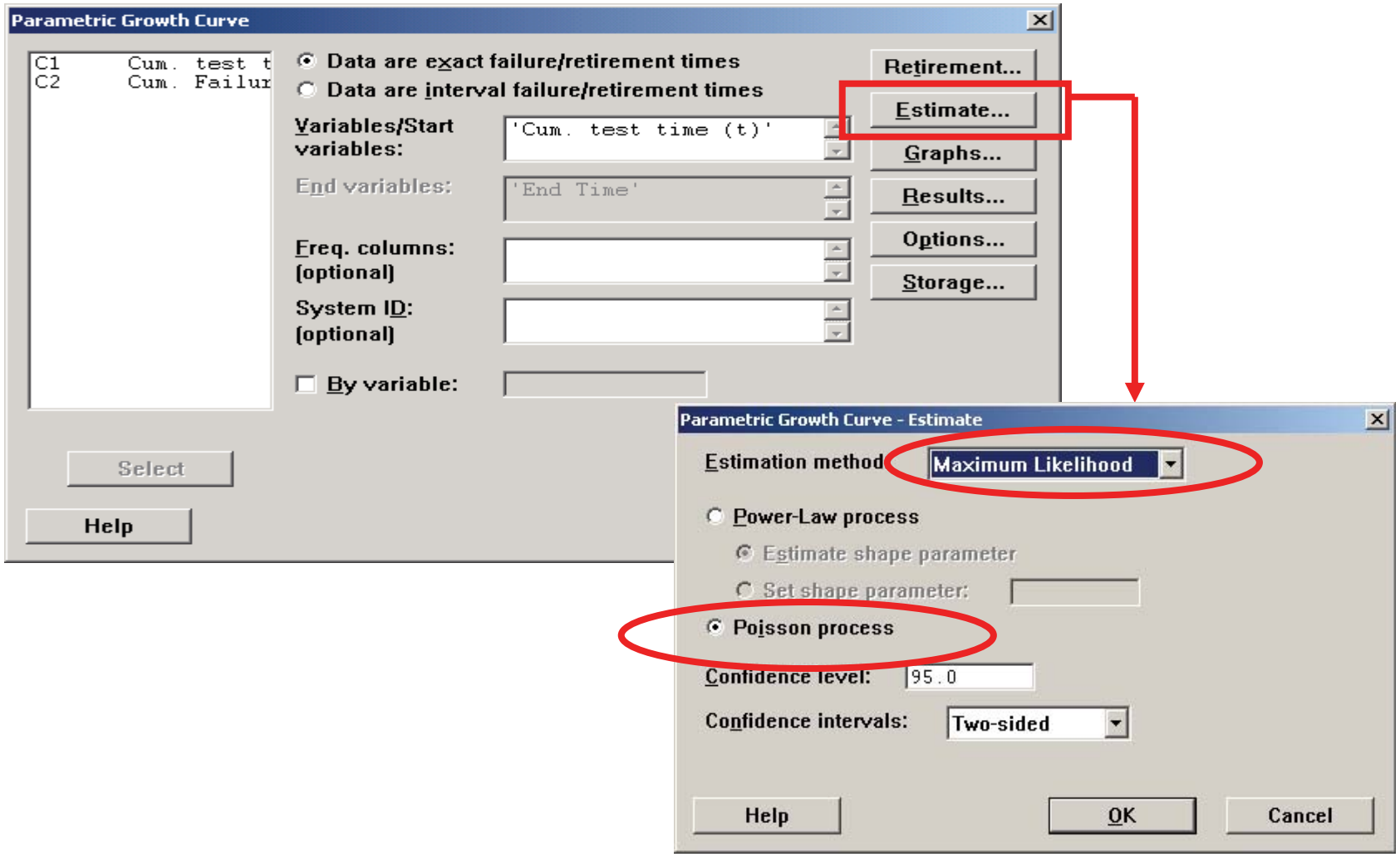
Since β (0.840) is less than 1, This implies that:

Failure Rate has been decreasing

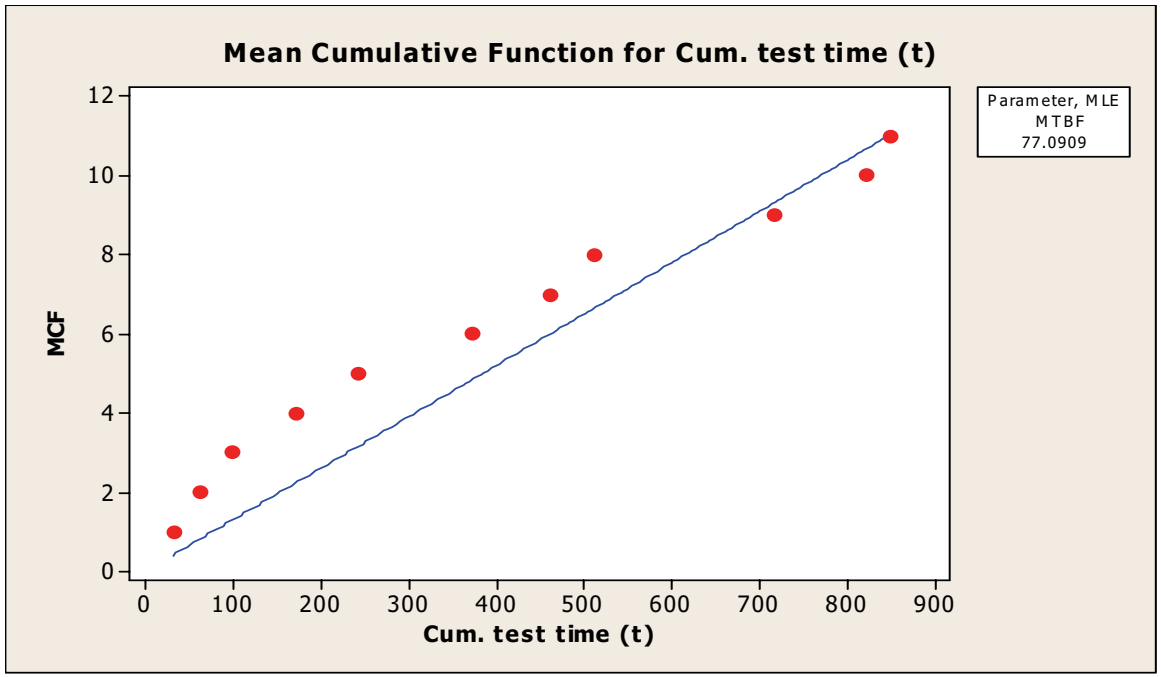
i.e.,

MTBF has been increasing

Minitab – Options



MTBF Estimation



Parameter Estimate

Parameter	Estimate	Standard Error	95% Normal CI Lower	Upper
MTBF	77.0909	23.244	42.6930	139.203