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**Electrical insulating materials –
Thermal endurance properties –**

**Part 3:
Instructions for calculating thermal
endurance characteristics**



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IEC 60216-3

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Electrical insulating materials – Thermal endurance properties –

Part 3: Instructions for calculating thermal endurance characteristics

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INTERNATIONAL ELECTROTECHNICAL COMMISSION

ELECTRICAL INSULATING MATERIALS – THERMAL ENDURANCE PROPERTIES –

Part 3: Instructions for calculating thermal endurance characteristics

FOREWORD

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International Standard IEC 60216-3 has been prepared by IEC technical committee 112: Evaluation and qualification of electrical insulating materials and systems¹.

This second edition of IEC 60216-3 cancels and replaces the first edition, published in 2002, and constitutes a technical revision.

The major technical changes with regard to the first edition concern an updating of Table C.2. In addition, the scope has been extended to cover a greater range of data characteristics, particularly with regard to incomplete data, as often obtained from proof test criteria. The greater flexibility of use should lead to more efficient employment of the time available for ageing purposes. Finally, the procedures specified in this part of IEC 60216 have been extensively tested and have been used to calculate results from a large body of experimental data obtained in accordance with other parts of the standard. Annex E "Computer program" has been completely reworked.

¹ Provisional title: IEC technical committee 112 has been formed out of a merger between subcommittee 15E and technical committee 98.

The text of this standard is based on the following documents:

FDIS	Report on voting
112/26/FDIS	112/29/RVD

Full information on the voting for the approval of this standard can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

IEC 60216 consists of the following parts, under the general title *Electrical insulating materials – Thermal endurance properties*²:

- Part 1: Ageing procedures and evaluation of test results
- Part 2: Determination of thermal endurance properties of electrical insulating materials – Choice of test criteria
- Part 3: Instructions for calculating thermal endurance characteristics
- Part 4: Ageing ovens
- Part 5: Determination of relative thermal endurance index (RTE) of an insulating material
- Part 6: Determination of thermal endurance indices (TI and RTE) of an insulating material using the fixed time frame method

NOTE This series may be extended. For revisions and new parts, see the current catalogue of IEC publications for an up-to-date list.

The committee has decided that the contents of this publication will remain unchanged until the maintenance result date indicated on the IEC web site under "<http://webstore.iec.ch>" in the data related to the specific publication. At this date, the publication will be

- reconfirmed;
- withdrawn;
- replaced by a revised edition, or
- amended.

A CD-ROM containing the computer program and data files referred to in Annex E is affixed to the back cover of this publication.

A bilingual version of this publication may be issued at a later date.

² Titles of existing parts in this series will be updated at the time of their next revision.

ELECTRICAL INSULATING MATERIALS – THERMAL ENDURANCE PROPERTIES –

Part 3: Instructions for calculating thermal endurance characteristics

1 Scope

This part of IEC 60216 specifies the calculation procedures to be used for deriving thermal endurance characteristics from experimental data obtained in accordance with the instructions of IEC 60216-1 and IEC 60216-2, using fixed ageing temperatures and variable ageing times.

The experimental data may be obtained using non-destructive, destructive or proof tests. Data obtained from non-destructive or proof tests may be incomplete, in that measurement of times taken to reach the endpoint may have been terminated at some point after the median time but before all specimens have reached end-point.

The procedures are illustrated by worked examples, and suitable computer programs are recommended to facilitate the calculations.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60216-1:2001, *Electrical insulating materials – Properties of thermal endurance – Part 1: Ageing procedures and evaluation of test results*

IEC 60216-2:2005, *Electrical insulating materials – Properties of thermal endurance – Part 2: Determination of thermal endurance properties of electrical insulating materials – Choice of test criteria*

IEC 60493-1:1974, *Guide for the statistical analysis of ageing test data – Part 1: Methods based on mean values of normally distributed test results*

3 Terms, definitions, symbols and abbreviated terms

3.1 Terms and definitions

For the purposes of this document, the following definitions apply.

3.1.1

ordered data

group of data arranged in sequence so that in the appropriate direction through the sequence each member is greater than, or equal to, its predecessor

NOTE 1 In this standard, ascending order implies that the data is ordered in this way, the first being the smallest.

NOTE 2 It has been established that the term “group” is used in the theoretical statistics literature to represent a subset of the whole data set. The group comprises those data having the same value of one of the parameters of the set (e.g. ageing temperature). A group may itself comprise a number of sub-groups characterised by another parameter (e.g. time in the case of destructive tests).

3.1.2**order-statistic**

each individual value in a group of ordered data is referred to as an order-statistic identified by its numerical position in the sequence

3.1.3**incomplete data**

ordered data, where the values above and/or below defined points are not known

3.1.4**censored data**

incomplete data, where the number of unknown values is known.

NOTE 1 If the censoring is begun above/below a specified numerical value, the censoring is Type I. If above/below a specified order-statistic it is Type II This standard is concerned only with Type II.

3.1.5**degrees of freedom**

number of data values minus the number of parameter values

3.1.6**variance of a data group**

sum of the squares of the deviations of the data from a reference level.

NOTE 1 The reference level may be defined by one or more parameters, for example a mean value (one parameter) or a line (two parameters, slope and intercept), divided by the number of degrees of freedom

3.1.7**central second moment of a data group**

sum of the squares of the differences between the data values and the value of the group mean, divided by the number of data in the group

3.1.8**covariance of data groups**

for two groups of data with equal numbers of elements where each element in one group corresponds to one in the other, the sum of the products of the deviations of the corresponding members from their group means, divided by the number of degrees of freedom

3.1.9**regression analysis**

process of deducing the best-fit line expressing the relation of corresponding members of two data groups by minimizing the sum of squares of deviations of members of one of the groups from the line

NOTE The parameters are referred to as the regression coefficients.

3.1.10**correlation coefficient**

number expressing the completeness of the relation between members of two data groups, equal to the covariance divided by the square root of the product of the variances of the groups

NOTE The value of its square is between 0 (no correlation) and 1 (complete correlation).

3.1.11**end-point line**

line parallel to the time axis intercepting the property axis at the end-point value

3.2 Symbols and abbreviated terms

		Subclause
a	Regression coefficient (y -intercept)	4.3, 6.2
a_p	Regression coefficient for destructive test calculations	6.1
b	Regression coefficient (slope)	4.3, 6.2
b_p	Regression coefficient for destructive test calculations	6.1
b_r	Intermediate constant (calculation of \hat{X}_c)	6.3
c	Intermediate constant (calculation of χ^2)	6.3
f	Number of degrees of freedom	Tables C.2 to C5
F	Fisher distributed stochastic variable	4.2, 6.1, 6.3
F_0	Tabulated value of F (linearity of thermal endurance graph)	4.4, 6.3
F_1	Tabulated value of F (linearity of property graph – significance 0,05)	6.1
F_2	Tabulated value of F (linearity of property graph – significance 0,005)	6.1
g	Order number of ageing time for destructive tests	6.1
h	Order number of property value for destructive tests	6.1
HIC	Halving interval at temperature equal to T_I	4.3, 7
HIC _{g}	Halving interval corresponding to T_{I_g}	7.3
i	Order number of exposure temperature	4.1, 6.2
j	Order number of time to end-point	4.1, 6.2
k	Number of ageing temperatures	4.1, 6.2
m_i	Number of specimens aged at temperature ϑ_i	4.1, 6.1
N	Total number of times to end-point	6.2
n_g	Number of property values in group aged for time τ_g	6.1
n_i	Number of values of y at temperature ϑ_i	4.1, 6.1
\bar{p}	Mean value of property values in selected groups	6.1
p	Value of diagnostic property	6.1
P	Significance level of χ^2 distribution	4.4, 6.3.1
p_e	Value of diagnostic property at end-point for destructive tests	6.1
\bar{p}_g	Mean of property values in group aged for time τ_g	6.1
p_{gh}	Individual property value	6.1
q	Base of logarithms	6.3
r	Number of ageing times selected for inclusion in calculation (destructive tests)	6.1
r^2	Square of correlation coefficient	6.2.3
s^2	Weighted mean of s_1^2 and s_2^2	6.3
s_1^2	Weighted mean of s_{ij}^2 , pooled variance within selected groups	4.3, 6.1 - 6.3
$(s_1^2)_a$	Adjusted value of s_1^2	4.4, 6.3
s_{1g}^2	Variance of property values in group aged for time τ_g	6.1
s_{ii}^2	Variance of y_{ij} values at temperature ϑ_i	4.3, 6.2
s_2^2	Variance about regression line	6.1 - 6.3
s_a^2	Adjusted value of s^2	6.3

s_r^2	Intermediate constant	6.3
s_Y^2	Variance of Y	6.3
t	Student distributed stochastic variable	6.3
t_c	Adjusted value of t (incomplete data)	6.3
TC	Lower 95 % confidence limit of TI	4.4, 7
TC _a	Adjusted value of TC	7.1
TI	Temperature Index	4.3, 7
TI ₁₀	Temperature Index at 10 kh	7.1
TI _a	Adjusted value of TI	7.3
TI _g	Temperature index obtained by graphical means or without defined confidence limits	7.3
x	Independent variable: reciprocal of thermodynamic temperature	
\bar{x}	Weighted mean value of x	6.2
X	Specified value of x for estimation of y	6.3
\hat{X}	Estimated value of x at specified value of y	6.3
\hat{X}_c	Upper 95 % confidence limit of \hat{X}	6.3
x_i	Reciprocal of thermodynamic temperature corresponding to ϑ_i	4.1, 6.1
\bar{y}	Weighted mean value of y	6.2
y	Dependent variable: logarithm of time to end-point	
\hat{Y}	Estimated value of y at specified value of x	6.3
Y	Specified value of y for estimation of x	6.3
\hat{Y}_c	Lower 95 % confidence limit of \hat{Y}	6.3
\bar{y}_i	Mean values of y_{ij} at temperature ϑ_i	4.3, 6.2
y_{ij}	Value of y corresponding to τ_{ij}	4.1, 6.1
\bar{z}	Mean value of z_g	6.1
z_g	Logarithm of ageing time for destructive tests – group g	6.1
α	Censored data coefficient for variance	4.3, 6.2
β	Censored data coefficient for variance	4.3, 6.2
ε	Censored data coefficient for variance of mean	4.3, 6.2
Θ_0	The temperature 0 °C on the thermodynamic scale (273,15 K)	4.1, 6.1
$\hat{\vartheta}$	Estimate of temperature for temperature index	6.3.3
$\hat{\vartheta}_c$	Confidence limit of $\hat{\vartheta}$	6.3.3
ϑ_i	Ageing temperature for group i	4.1, 6.1
μ	Censored data coefficient for mean	4.3, 6.2
$\mu_2(x)$	Central second moment of x values	6.2, 6.3
ν	Total number of property values selected at one ageing temperature	6.1
τ_f	Time selected for estimate of temperature	6.3
τ_{ij}	Times to end-point	6.4
χ^2	χ^2 -distributed stochastic variable	6.3

4 Principles of calculations

4.1 General principles

The general calculation procedures and instructions given in Clause 6 are based on the principles set out in IEC 60493-1. These may be simplified as follows (see 3.7.1 of IEC 60493-1:1974):

- a) the relation between the mean of the logarithms of the times taken to reach the specified end-point (times to end-point) and the reciprocal of the thermodynamic (absolute) temperature is linear;
- b) the values of the deviations of the logarithms of the times to end-point from the linear relation are normally distributed with a variance which is independent of the ageing temperature.

The data used in the general calculation procedures are obtained from the experimental data by a preliminary calculation. The details of this calculation are dependent on the character of the diagnostic test: non-destructive, proof or destructive (see 4.2). In all cases the data comprise values of x , y , m , n and k

where

$x_i = 1/(\vartheta_i + \Theta_0)$ is the reciprocal of thermodynamic value of ageing temperature ϑ_i in °C;

$y_{ij} = \log \tau_{ij}$ is the logarithm of value of time (j) to end-point at temperature ϑ_i ;

n_i is the number of y values in group number i aged at temperature ϑ_i ;

m_i is the number of samples in group number i aged at temperature ϑ_i (different from n_i for censored data);

k is the number of ageing temperatures or groups of y values.

NOTE Any number may be used as the base for logarithms, provided consistency is observed throughout calculations. The use of natural logarithms (base e) is recommended, since most computer programming languages and scientific calculators have this facility.

4.2 Preliminary calculations

In all cases, the reciprocals of the thermodynamic values of the ageing temperatures are calculated as the values of x_i .

The values of y_{ij} are calculated as the values of the logarithms of the individual times to end-point τ_{ij} obtained as described below.

In many cases of non-destructive and proof tests, it is advisable for economic reasons, (for example, when the scatter of the data is high) to stop ageing before all specimens have reached the end-point, at least for some temperature groups. In such cases, the procedure for calculation on censored data (see 6.2.1.2) shall be carried out on the (x , y) data available.

Groups of complete and incomplete data or groups censored at a different point for each ageing temperature may be used together in one calculation in 6.2.1.2.

4.2.1 Non-destructive tests

Non-destructive tests (for example, loss of mass on ageing) give directly the value of the diagnostic property of each specimen each time it is measured at the end of an ageing period. The time to end-point τ_{ij} is therefore available, either direct or by linear interpolation between consecutive measurements.

4.2.2 Proof tests

The time to end-point t_{ij} for an individual specimen is taken as the mid-point of the ageing period immediately prior to reaching the end-point (6.3.2 of IEC 60216-1:2001).

4.2.3 Destructive tests

When destructive test criteria are employed, each test specimen is destroyed in obtaining a property value and its time to end-point cannot therefore be measured directly.

To enable estimates of the times to end-point to be obtained, the assumptions are made that in the vicinity of the endpoint:

- the relation between the mean property values and the logarithm of the ageing time is approximately linear;
- the values of the deviations of the individual property values from this linear relation are normally distributed with a variance which is independent of the ageing time;
- the curves of property versus logarithm of time for the individual test specimens are straight lines parallel to the line representing the relation of a) above.

For application of these assumptions, an ageing curve is drawn for the data obtained at each of the ageing times. The curve is obtained by plotting the mean value of property for each specimen group against the logarithm of its ageing time. If possible, ageing is continued at each temperature until at least one group mean is beyond the end-point level. An approximately linear region of this curve is drawn in the vicinity of the end-point line (see Figure D.2).

A statistical test (F -test) is carried out to decide whether deviations from linearity of the selected region are acceptable (see 6.1.4.4). If acceptable, then, on the same graph, points representing the properties of the individual specimens are drawn. A line parallel to the ageing line is drawn through each individual specimen data point. The estimate of the logarithm of the time to end-point for that specimen (y_{ij}) is then the value of the logarithm of time corresponding to the intersection of the line with the end-point line (Figure D.2).

With some limitations, an extrapolation of the linear mean value graph to the end-point level is permitted.

The above operations are executed numerically in the calculations detailed in 6.1.4.

4.3 Variance calculations

Commencing with the values of x and y obtained as above, the following calculations are made:

For each group of y_{ij} values, the mean \bar{y}_i and variance s_{i1}^2 are calculated, and from the latter the pooled variance within the groups, s_1^2 , is derived, weighting the groups according to size.

For incomplete data, the calculations have been developed from those originated by Saw [1]¹ and given in 6.2.1.2. The coefficients required (μ for mean, α , β for variance and ε for deriving the variance of mean from the group variance) are given in Table C.1. For multiple groups, the variances are pooled, weighting according to the group size. The mean value of the group values of ε is obtained without weighting, and multiplied by the pooled variance.

NOTE The weighting according to the group size is implicit in the definition of ε , which here is equal to that originally proposed by Saw, multiplied by the group size. This makes for simpler representation in equations.

¹ Figures in square brackets refer to the bibliography.

From the means \bar{y}_i and the values of x_i , the coefficients a and b (the coefficients of the best fit linear representation of the relationship between x and y) are calculated by linear regression analysis.

From the regression coefficients, the values of TI and HIC are calculated. The variance of the deviations from the regression line is calculated from the regression coefficients and the group means.

4.4 Statistical tests

The following statistical tests are made:

- a) Fisher test for linearity (Fisher test, F -test) on destructive test data prior to the calculation of estimated times to end-point (see 4.2.3);
- b) variance equality (Bartlett's χ^2 -test) to establish whether the variances within the groups of y values differ significantly;
- c) F -test to establish whether the ratio of the deviations from the regression line to the pooled variance within the data groups is greater than the reference value F_0 , i.e. to test the validity of the Arrhenius hypothesis as applied to the test data.

In the case of data of very small dispersion, it is possible for a non-linearity to be detected as statistically significant which is of little practical importance.

In order that a result may be obtained even where the requirements of the F -test are not met for this reason, a procedure is included as follows:

- 1) increase the value of the pooled variance within the groups (s_1^2) by the factor F/F_0 so that the F -test gives a result which is just acceptable (see 6.3.2);
- 2) use this adjusted value $(s_1^2)_a$ to calculate the lower confidence limit TC_a of the result;
- 3) if the lower confidence interval $(TI - TC_a)$ is found acceptable, the non-linearity is deemed to be of no practical importance (see 6.3.2);
- 4) from the components of the data dispersion, (s_1^2) and (s_2^2) the confidence interval of an estimate is calculated using the regression equation.

When the temperature index (TI), its lower confidence limit (TC) and the halving interval (HIC) have been calculated, (see 7.1), the result is considered acceptable if

$$TI - TC \leq 0,6 \text{ HIC} \quad (1)$$

When the lower confidence interval $(TI - TC)$ exceeds 0,6 HIC by a small margin, a usable result may still be obtained, provided $F \leq F_0$, by substituting $(TC + 0,6 \text{ HIC})$ for the value of TI (see Clause 7).

4.5 Results

The temperature index (TI), its halving interval (HIC) and its lower 95 % confidence limit (TC) are calculated from the regression equation, making allowance as described above for minor deviations from the prescribed results of the statistical tests.

The mode of reporting of the temperature index and halving interval is determined by the results of the statistical tests (see 7.2).

It is necessary to emphasize the need to present the thermal endurance graph as part of the report, since a single numerical result, TI (HIC), cannot present an overall qualitative view of the test data, and appraisal of the data cannot be complete without this.

5 Requirements and recommendations for valid calculations

5.1 Requirements for experimental data

The data submitted to the procedures of this standard shall conform to the requirements of 5.1 to 5.8 of IEC 60216-1:2001.

5.1.1 Non-destructive tests

For most diagnostic properties in this category, groups of five specimens will be adequate. However, if the data dispersion (confidence interval, see 6.3.3) is found to be too great, more satisfactory results are likely to be obtained by using a greater number of specimens. This is particularly true if it is necessary to terminate ageing before all specimens have reached end-point.

5.1.2 Proof tests

Not more than one specimen in any group shall reach end-point during the first ageing period: if more than one group contains such a specimen, the experimental procedure should be carefully examined (see 6.1.3) and the occurrence included in the test report.

The number of specimens in each group shall be at least five, and for practical reasons the maximum number treatable is restricted to 31 (Table C.1). The recommended number for most purposes is 21.

5.1.3 Destructive tests

At each temperature, ageing should be continued until the property value mean of at least one group is above and at least one below the end-point level. In some circumstances, and with appropriate limitations, a small extrapolation of the property value mean past the end-point level may be permitted (see 6.1.4.4). This shall not be permitted for more than one temperature group.

5.2 Precision of calculations

Many of the calculation steps involve summing of the differences of numbers or the squares of these differences, where the differences may be small by comparison with the numbers. In these circumstances it is necessary that the calculations be made with an internal precision of at least six significant digits, and preferably more, to achieve a result precision of three significant digits. In view of the repetitive and tedious nature of the calculations, it is strongly recommended that they be performed using a programmable calculator or microcomputer, in which case internal precision of ten or more significant digits is easily available.

6 Calculation procedures

6.1 Preliminary calculations

6.1.1 Temperatures and x -values

For all types of test, express each ageing temperature in K on the thermodynamic temperature scale, and calculate its reciprocal for use as x_i :

$$x_i = 1/(T_i + \Theta_0) \quad (2)$$

where $\Theta_0 = 273,15$ K.

6.1.2 Non-destructive tests

For specimen number j of group number i a property value after each ageing period is obtained. From these values, if necessary by linear interpolation, obtain the time to end-point and calculate its logarithm as y_{ij} .

6.1.3 Proof tests

For specimen number j of group number i calculate the mid point of the ageing period immediately prior to reaching the end-point and take the logarithm of this time as y_{ij} .

A time to end-point within the first ageing period shall be treated as invalid. Either:

- a) start again with a new group of specimens, or
- b) ignore the specimen and reduce the value ascribed to the number of specimens in the group (m_i) by one in the calculation for group means and variances (see 6.2.1.2).

If the end-point is reached for more than one specimen during the first period, discard the group and test a further group, paying particular attention to any critical points of experimental procedure.

6.1.4 Destructive tests

Within the groups of specimens aged at each temperature ϑ_i , carry out the procedures described in 6.1.4.1 to 6.1.4.5.

NOTE The subscript i is omitted from the expressions in 6.1.4.2 to 6.1.4.4 in order to avoid confusing multiple subscript combinations in print. The calculations of these subclauses shall be carried out separately on the data from each ageing temperature.

6.1.4.1 Calculate the mean property value for the data group obtained at each ageing time and the logarithm of the ageing time. Plot these values on a graph with the property value p as ordinate and the logarithm of the ageing time z as abscissa (see Figure D.2). Fit by visual means a smooth curve through the mean property points.

6.1.4.2 Select a time range within which the curve so fitted is approximately linear (see 6.1.4.4). Ensure that this time range includes at least three mean property values with at least one point on each side of the end-point line $p = p_e$. If this is not the case, and further measurements at greater times cannot be made (for example, because no specimens remain), a small extrapolation is permitted, subject to the conditions of 6.1.4.4.

Let the number of selected mean values (and corresponding value groups) be r , the logarithms of the individual ageing times be z_g and the individual property values be p_{gh} , where

$g = 1 \dots r$ is the order number of the selected group tested at time τ_g ;

$h = 1 \dots n_g$ is the order number of the property value within group number g ;

n_g is the number of property values in group number g .

In most cases, the number n_g of specimens tested at each test time is identical, but this is not a necessary condition, and the calculation can be carried out with different values of n_g for different groups.

Calculate the mean value \bar{p}_g and the variance s_{1g}^2 for each selected property value group.

$$\bar{p}_g = \sum_{h=1}^{n_g} p_{gh} / n_g \quad (3)$$

$$s_{1g}^2 = \left(\sum_{h=1}^{n_g} p_{gh}^2 - n_g \bar{p}_g^2 \right) / (n_g - 1) \quad (4)$$

Calculate the logarithms of τ_g :

$$z_g = \log \tau_g \quad (5)$$

6.1.4.3 Calculate the values

$$v = \sum_{g=1}^r n_g \quad (6)$$

$$\bar{z} = \sum_{g=1}^r z_g n_g / v \quad (7)$$

$$\bar{p} = \sum_{g=1}^r \bar{p}_g n_g / v \quad (8)$$

Calculate the coefficients of the regression equation $p = a_p + b_p z$

$$b_p = \frac{\left(\sum_{g=1}^r n_g z_g \bar{p}_g - v \bar{z} \bar{p} \right)}{\left(\sum_{g=1}^r n_g z_g^2 - v \bar{z}^2 \right)} \quad (9)$$

$$a_p = \bar{p} - b_p \bar{z} \quad (10)$$

Calculate the pooled variance within the property groups

$$s_1^2 = \sum_{g=1}^r (n_g - 1) s_{1g}^2 / (v - r) \quad (11)$$

Calculate the weighted variance of the deviations of the property group means from the regression line

$$s_2^2 = \sum_{g=1}^r n_g (\bar{p}_g - \hat{p}_g)^2 / (r - 2) \quad (12)$$

where

$$\hat{p}_g = a_p + b_p z_g \quad (13)$$

This may also be expressed as

$$s_2^2 = \left[\left(\sum_{g=1}^r n_g \bar{p}_g^2 - v \bar{p}^2 \right) - b_P \left(\sum_{g=1}^r n_g z_g \bar{p}_g - v \bar{z} \bar{p} \right) \right] / (r-2) \quad (14)$$

6.1.4.4 Make the *F*-test for non-linearity at significance level 0,05 by calculating

$$F = s_2^2 / s_1^2 \quad (15)$$

If the calculated value of *F* exceeds the tabulated value *F*₁ with *f*_n = *r* - 2 and *f*_d = *v* - *r* degrees of freedom (see Table C.2).

$$F_1 = F(0,95, r - 2, v - r)$$

change the selection in 6.1.4.2 and repeat the calculations.

If it is not possible to satisfy the *F*-test on the significance level 0,05 with *r* ≥ 3, make the *F*-test at a significance level 0,005 by comparing the calculated value of *F* with the tabulated value *F*₂ with *f*_n = *r* - 2 and *f*_d = *v* - *r* degrees of freedom (see Table C.3).

$$F_2 = F(0,995, r - 2, v - r)$$

If the test is satisfied at this level, the calculations may be continued, but the adjustment of TI according to 7.3.2 is not permitted.

If the *F*-test on significance level 0,005 (i.e. *F* ≤ *F*₂) cannot be satisfied, or the property points plotted according to 6.1.4.1 are all on the same side of the end-point line, an extrapolation may be permitted, subject to the following condition.

If the *F*-test on significance level 0,05 can be met for a range of values (with *r* ≥ 3) where all mean values \bar{p}_g are on the same side of the end-point value *p_e*, an extrapolation may be made provided that the absolute value of the difference between the end-point value *p_e* and the mean value \bar{p}_g closest to the end-point (usually \bar{p}_r) is less than 0,25 of the absolute value of the difference ($\bar{p}_1 - \bar{p}_r$).

In this case, calculations can be continued, but again it is not permitted to carry out the adjustment of TI according to 7.3.2.

6.1.4.5 For each value of property in each of the selected groups, calculate the logarithm of the estimated time to end-point:

$$y_{ij} = z_g - (p_{gh} - p_e) / b_P \quad (16)$$

$$n_i = v \quad (17)$$

where

j = 1 ... *n_i* is the order number of the *y*-value in the group of estimated *y*-values at temperature

t_i and *z_g* are the logarithms of the ageing time.

The *n_i* values of *y_{ij}* are the log(time) values to be used in the calculations of 6.2.1.

6.1.5 Incomplete data

In the case of incomplete data, arrange each group of y values in ascending order (see 3.1.1).

6.2 Main calculations

6.2.1 Calculation of group means and variances

Calculate the mean and variance of the group of y -values, y_{ij} , obtained at each temperature ϑ_i .

6.2.1.1 Complete data

For tests where the data are complete (i.e. not censored) the conventional equations may be used:

$$\bar{y}_i = \sum_{j=1}^{n_i} y_{ij} / n_i \quad (18)$$

$$s_{ii}^2 = \left(\sum_{j=1}^{n_i} y_{ij}^2 - n_i \bar{y}_i^2 \right) / (n_i - 1) \quad (19)$$

Alternatively, the equations for incomplete data (6.2.1.2) may be used, although they are much less convenient for this purpose. The coefficients are then given the following values:

$$\alpha_i = 1 / (n_i - 1) \quad (20)$$

$$\beta_i = \frac{-1}{n_i (n_i - 1)} \quad (21)$$

$$\mu_i = 1 - 1 / n_i \quad (22)$$

NOTE These expressions are derived by simple algebra. If the expression for mean or variance (see equations (18) and (19)) is equated to that obtained using equations (23) and (24), the single unknown in each resulting equation can be made the subject of the equation, resulting in the expressions of equations (20) to (22). The value of ε is obviously 1.

6.2.1.2 Censored data

Instead of Equations (18) and (19), the following equations shall be used:

$$\bar{y}_i = (1 - \mu_i) y_{in_i} + \mu_i \sum_{j=1}^{n_i-1} \frac{y_{ij}}{(n_i - 1)} \quad (23)$$

$$s_{ii}^2 = \alpha_i \sum_{j=1}^{n_i-1} (y_{in_i} - y_{ij})^2 + \beta_i \left[\sum_{j=1}^{n_i-1} (y_{in_i} - y_{ij}) \right]^2 \quad (24)$$

The values of μ_i , α_i , and β_i shall be read from the appropriate lines of Table C.1. Where data are partially censored (i.e. one or more temperature groups is complete and one or more censored) the values shall be derived using Equations (20) to (22).

6.2.2 General means and variances

Calculate the total number of y_{ij} values, N , the weighted mean value of x , (\bar{x}) , and the weighted mean value of y , (\bar{y}) :

$$N = \sum_{i=1}^k n_i \quad (25)$$

$$\bar{x} = \sum_{i=1}^k n_i x_i / N \quad (26)$$

$$\bar{y} = \sum_{i=1}^k n_i \bar{y}_i / N \quad (27)$$

For censored data, calculate the total number of test specimens:

$$M = \sum_{i=1}^k m_i \quad (28)$$

For complete data, $M = N$.

For censored data, read the values of ε_i from Table C.1. For complete data, or if $n_i = m_i$ in partially censored data, the value of ε_i shall be 1.

Calculate the general mean variance factor:

$$\varepsilon = \sum_{i=1}^k \varepsilon_i / k \quad (29)$$

Calculate the pooled variance within the data groups:

$$s_1^2 = \varepsilon \sum_{i=1}^k (n_i - 1) s_{ii}^2 / (N - k) \quad (30)$$

Calculate the second central moment of the x values:

$$\mu_2(x) = \frac{\left(\sum_{i=1}^k n_i x_i^2 - N \bar{x}^2 \right)}{N} \quad (31)$$

6.2.3 Regression calculations

In the expression for the regression line:

$$y = a + bx \quad (32)$$

Calculate the slope:

$$b = \frac{\left(\sum_{i=1}^k n_i x_i \bar{y}_i - N \bar{x} \bar{y} \right)}{\left(\sum_{i=1}^k n_i x_i^2 - N \bar{x}^2 \right)} \quad (33)$$

the intercept on the y -axis

$$a = \bar{y} - b \bar{x} \quad (34)$$

and the square of the correlation coefficient:

$$r^2 = \frac{\left(\sum_{i=1}^k n_i x_i \bar{y}_i - N \bar{x} \bar{y} \right)^2}{\left(\sum_{i=1}^k n_i x_i^2 - N \bar{x}^2 \right) \left(\sum_{i=1}^k n_i y_i^2 - N \bar{y}^2 \right)} \quad (35)$$

Calculate the variance of the deviations of the y -means from the regression line:

$$s_2^2 = \sum_{i=1}^k \frac{n_i (\bar{y}_i - \hat{Y}_i)^2}{(k-2)}, \quad \hat{Y}_i = a + b x_i \quad (36)$$

or

$$s_2^2 = \frac{(1-r^2)}{(k-2)} \left(\sum_{i=1}^k n_i \bar{y}_i^2 - N \bar{y}^2 \right) \quad (37)$$

6.3 Statistical tests

6.3.1 Variance equality test

Calculate the value of Bartlett's χ^2 function:

$$\chi^2 = \frac{\ln q}{c} \left[(N-k) \log_q \frac{s_1^2}{\varepsilon} - \sum_{i=1}^k (n_i - 1) \log_q s_{1i}^2 \right] \quad (38)$$

where

$$c = 1 + \frac{\left(\sum_{i=1}^k \frac{1}{n_i - 1} - \frac{1}{N - k} \right)}{3(k-1)} \quad (39)$$

q is the base of the logarithms used in this equation. It need not be the same as that used in the calculations elsewhere in this clause.

If $q = 10$, $\ln q = 2,303$, if $q = e$, $\ln q = 1$.

Compare the value of χ^2 with the tabulated value for $f = (k - 1)$ degrees of freedom (Table C.5). If the value of χ^2 is greater than the value tabulated for a significance level of 0,05, report the value of χ^2 and the significance level tabulated for the highest value less than χ^2 . Alternatively, if both χ^2 and its significance level are calculated by a computer program, report these.

6.3.2 Linearity test (F -test)

The variance of the deviations from the regression line s_2^2 is compared with the pooled variance within the k groups of measurements s_1^2 by the F -test at a significance level of 0,05.

Calculate the ratio

$$F = s_2^2 / s_1^2 \quad (40)$$

and compare its value with the tabulated value F_0 with $f_n = k - 2$ and $f_d = N - k$ degrees of freedom (Tables C.2 and C.3).

$$F_0 = F(0,95, k - 2, N - k)$$

a) If $F \leq F_0$, calculate the pooled variance estimate

$$s^2 = \frac{(N - k)s_1^2 + (k - 2)s_2^2}{(N - 2)} \quad (41)$$

b) If $F > F_0$, adjust s_1^2 to $(s_1^2)_a = s_1^2(F / F_0)$ and calculate an adjusted value of s^2

$$s_a^2 = \frac{(N - k)(s_1^2)_a + (k - 2)s_2^2}{(N - 2)} \quad (42)$$

6.3.3 Confidence limits of X and Y estimates

Obtain the tabulated value of Student's t with $N - 2$ degrees of freedom at a confidence level of 0,95, $t_{0,95,N-2}$ (Table C.4).

Calculate the value of t (t_c) corrected for the amount of censoring of the data:

$$t_c = \left(\frac{1}{t_{0,95,N-2}} - \frac{(1 - N/M)}{(N/8 + 4,5)} \right)^{-1} \quad (43)$$

a) Y -estimates

Calculate the estimated value of Y corresponding to the given X and its lower 95 % confidence limit:

$$\hat{Y}_c = \hat{Y} - t_c s_Y, \quad \hat{Y} = a + bx \quad (44)$$

$$s_{\hat{Y}}^2 = \frac{s^2}{N} \left[1 + \frac{(X - \bar{x})^2}{\mu_2(x)} \right] \quad (45)$$

For the confidence limit curve of the thermal endurance graph (see 6.4), Y_c is calculated for several (X, Y) pairs of values over the range of interest, and the curve drawn through the points (X, Y_c) plotted on the graph.

If $F > F_0$ the value of s^2 shall be replaced by s_a^2 (Equation (42)).

b) X -estimates

Calculate the value of \hat{X} and its upper 95 % confidence limit, corresponding to a time to end-point τ_f :

$$\hat{X}_c = \bar{x} + \frac{(Y - \bar{y})}{b_r} + \frac{t_c s_r}{b_r} \quad (46)$$

$$Y = \log \tau_f \quad : \quad \hat{X} = (Y - a) / b \quad (47)$$

$$b_r = b - \frac{t_c^2 s^2}{N b \mu_2(x)} \quad (48)$$

$$s_r^2 = \frac{s^2}{N} \left(\frac{b_r}{b} + \frac{(\hat{X} - \bar{x})^2}{\mu_2(x)} \right) \quad (49)$$

The temperature estimate and its lower 95 % confidence limit shall be calculated from the corresponding X estimate and its upper confidence limit:

$$\hat{\vartheta} = \frac{1}{\hat{X}} - \Theta_0 \quad , \quad \hat{\vartheta}_c = \frac{1}{\hat{X}_c} - \Theta_0 \quad (50)$$

6.4 Thermal endurance graph

When the regression line has been established, it is drawn on the thermal endurance graph, i.e. a graph with $y = \log(\tau)$ as ordinate and $x = 1/(\vartheta + \Theta_0)$ as abscissa. Usually x is plotted as increasing from right to left and the corresponding values of ϑ in degrees Celsius ($^{\circ}\text{C}$) are marked on this axis (see Figures D.1a and D.1b). Special graph paper is obtainable for this purpose.

Alternatively, a computer programme executing this calculation may include a subroutine to plot the graph on the appropriate non-linear scale.

The individual values $y_{ij} = \log(\tau_{ij})$ and the mean values \bar{y}_i obtained as in 6.2.1 are plotted on the graph at the corresponding values of x_i :

$$x_i = 1/(\vartheta_i + \Theta_0) \quad (51)$$

The thermal endurance graph may be completed by drawing the lower 95 % confidence curve (see 6.3.3).

7 Calculation and requirements for results

7.1 Calculation of thermal endurance characteristics

Using the regression equation

$$y = a + bx \quad (52)$$

(the coefficients a and b being calculated according to 6.2.3) calculate the temperature in degrees Celsius ($^{\circ}\text{C}$) corresponding to a time to end-point of 20 kh. The numerical value of this temperature is the temperature index, TI.

Calculate by the same method the numerical value of the temperature corresponding to a time to end-point of 10 kh, TI_{10} . The halving interval HIC is:

$$HIC = TI_{10} - TI \quad (53)$$

Calculate by the method of 6.3.3 b), with $Y = \log 20000$, the lower 95 % confidence limit of TI : TC or TC_a if the adjusted value s_a^2 is used.

Determine the value of $(TI - TC)/HIC$ or $(TI - TC_a)/HIC$.

Plot the thermal endurance graph (see 6.4).

7.2 Summary of statistical tests and reporting

In Table B.1, if the condition in the column headed "Test" is not met, the action is as indicated in the final column. If the condition is met, the action is as indicated at the next step. The same sequence is indicated in the decision flow chart for thermal endurance calculations, see Annex A.

7.3 Reporting of results

7.3.1 If the value of $(TI - TC)/HIC$ is $\leq 0,6$, the test result shall be reported in the format

$$TI \text{ (HIC): } xxx \text{ (xx,x)} \quad (54)$$

in accordance with 6.8 of IEC 60216-1:2001.

7.3.2 If $0,6 < (TI - TC)/HIC \leq 1,6$ and at the same time, $F \leq F_0$ (see 6.3.2) the value

$$TI_a = TC + 0,6 \text{ HIC} \quad (55)$$

together with HIC shall be reported as $TI \text{ (HIC): } xxx \text{ (xx,x)}$

7.3.3 In all other cases the result shall be reported in the format

$$TI_g = \dots, \quad HIC_g = \dots \quad (56)$$

8 Test report

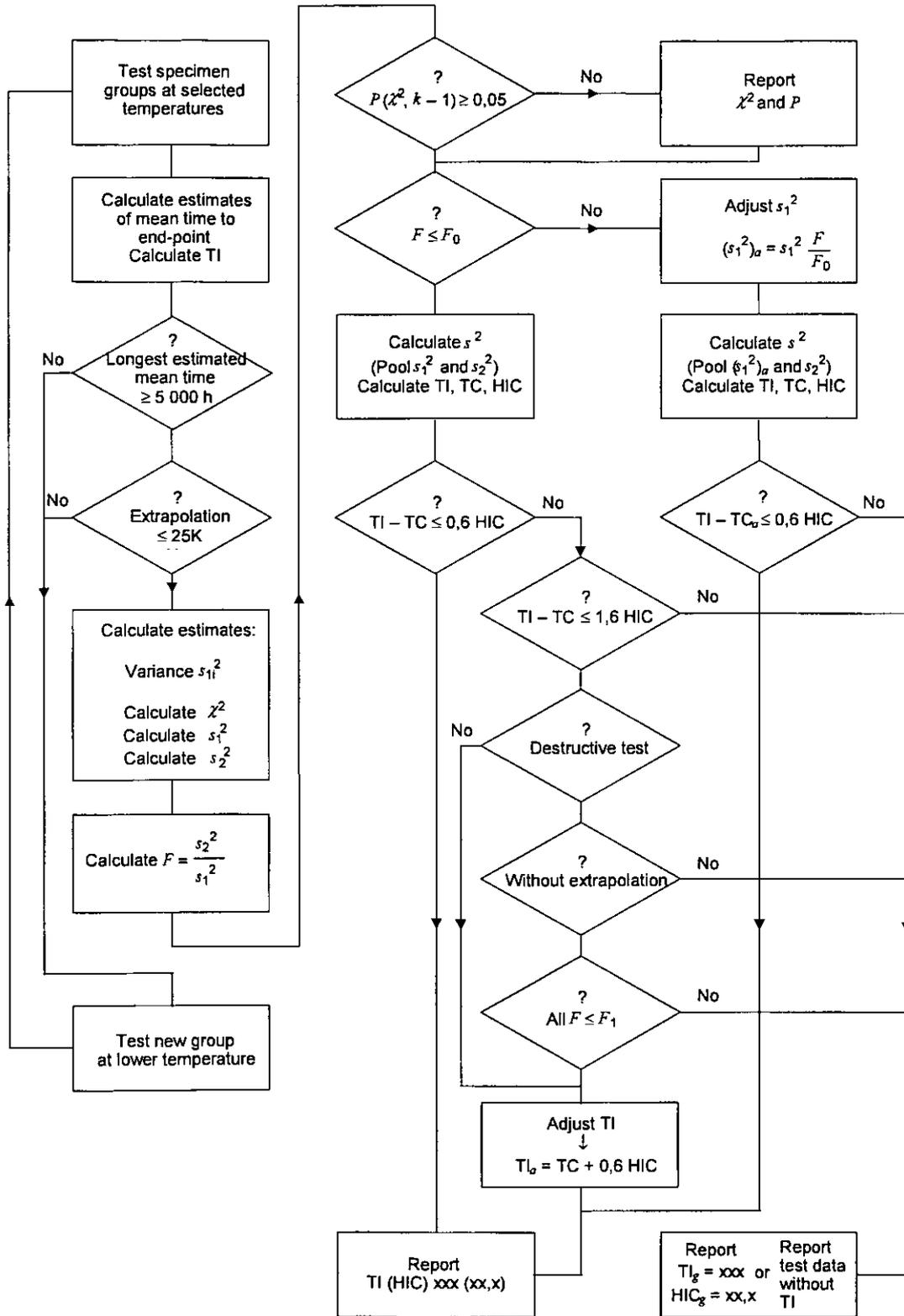
The test report shall include

- a) a description of the tested material including dimensions and any conditioning of the specimens;
- b) the property investigated, the chosen end-point, and, if it was required to be determined, the initial value of the property;
- c) the test method used for determination of the property (for example, by reference to an IEC publication);
- d) any relevant information on the test procedure, for example, ageing environment;
- e) the individual test temperatures, with the appropriate data for the test type;
 - 1) for non-destructive tests, the individual times to end-point;

- 2) for proof tests, the numbers and durations of the ageing cycles, with the numbers of specimens reaching end-point during the cycles;
 - 3) for destructive tests, the ageing times and individual property values, with the graphs of variation of property with ageing time;
- f) the thermal endurance graph;
 - g) the temperature index and halving interval reported in the format defined in 7.3;
 - h) the values of χ^2 and P if required by 6.3.1;
 - i) first-cycle failures in accordance with 5.1.2.

Annex A (normative)

Decision flow chart



Annex B (normative)

Decision table

Table B.1 – Decisions and actions according to tests

Step	Test or action ^a	Reference	Action if «NO» in test
1	Longest mean time to end-point $\geq 5\ 000$ h	5.5 of IEC 60216-1	Go to step 15
2	Extrapolation ≤ 25 K	5.5 of IEC 60216-1	Go to step 15
3	$P(\chi^2, f) \geq 0,05$	6.3.1	Report χ^2 and P Go to step 4
4	$F \leq F_0$	6.3.2	Go to step 12
5	$TI - TC \leq 0,6$ HIC	7.3	Go to step 7
6	Report TI (HIC): xxx (xx,x)	7.3	
7	$TI - TC \leq 1,6$ HIC	7.3	Go to step 14
8	Destructive test criteria used	6.1.4.4	Go to step 11
9	Were data processed without extrapolation	6.1.4.4	Go to step 14
10	Where all values of $F \leq F_1$	6.1.4.4	Go to step 14
11	Report $TI_a = TC + 0,6$ HIC as TI (HIC): ... (..)	7.3	
12	$TI - TC_a \leq 0,6$ HIC	6.3.2	Go to step 14
13	Report TI (HIC): xxx(xx,x)	7.3	
14	Report $TI_f = xxx$, $HIC_f = xx,x$	7.3	
15	Test new group at a lower temperature		

^a An action is indicated by bold print.

Annex C
(informative)

Statistical tables

Table C.1 – Coefficients for censored data calculations

m	n	α	β	μ	ϵ
5	3	614,4705061728	-100,3801985597	0,0000000000	860,4482888889
5	4	369,3153100012	-70,6712934899	472,4937150842	874,0745894447
6	4	395,4142139605	-58,2701183523	222,6915218468	835,7650306465
6	5	272,5287238052	-44,0988850936	573,5126123815	887,1066681426
7	4	415,5880351563	-46,5401552734	0,0000000000	841,7746734375
7	5	289,1914470089	-38,0060438107	364,2642153815	837,3681267819
7	6	215,5146796875	-30,1363662109	642,2345606152	898,7994404297
8	5	302,2559543304	-32,0455510095	173,7451925589	823,1325022970
8	6	227,1320334900	-26,7149242720	462,3946896558	845,5891673417
8	7	178,0192047851	-21,8909055649	692,0082911498	908,7175231765
9	5	312,9812000000	-26,3842700000	0,0000000000	830,5022000000
9	6	236,3858000000	-23,2986100000	296,0526300000	821,3172600000
9	7	186,6401000000	-19,7898900000	534,4601800000	855,2096700000
9	8	151,5120000000	-16,6140800000	729,7119900000	917,0583200000
10	6	244,1191560890	-20,0047740729	142,3739002847	815,8210886826
10	7	193,6205880047	-17,6663604814	386,9526017618	825,7590437753
10	8	158,2300608320	-15,2437931582	589,6341322307	864,6219294884
10	9	131,8030382363	-13,0347627976	759,2533663842	924,0989192531
11	6	250,6859320988	-16,8530354295	0,0000000000	822,9729127315
11	7	199,4695468487	-15,5836545374	249,2599953079	812,6308986254
11	8	163,6996121337	-13,8371182557	457,2090965743	832,5488161799
11	9	137,2299243827	-12,1001907793	633,2292924678	873,3355410880
11	10	116,5913210464	-10,4969569718	783,0177949444	930,0880372994
12	7	204,5349924229	-13,5767110244	120,5748554921	810,9803051840
12	8	168,3292196600	-12,4439880795	332,5519557674	814,7269021330
12	9	141,6425229674	-11,1219466676	513,1493415383	840,0625045817
12	10	121,0884792448	-9,8359507754	668,5392651269	881,2400322962
12	11	104,5060800375	-8,6333795848	802,5441292356	935,2282230049
13	7	208,9406118284	-11,6456142827	0,0000000000	817,5921863390
13	8	172,3464251400	-11,0865264201	215,2023355151	807,2699422973
13	9	145,4178687827	-10,1472348992	399,3236520338	819,3180095090
13	10	124,7371924225	-9,1300085328	558,7461589055	847,5908596926
13	11	108,3018058633	-8,1510819663	697,7158560873	888,3591181189
13	12	94,6796149706	-7,2252117874	818,8697028778	939,6794196639

NOTE α , β , μ and ϵ are all in units of 1×10^{-3} .

Table C.1 (continued)

m	n	α	β	μ	ϵ
14	8	175,9018422090	-9,7746826098	104,5543516980	807,5106793327
14	9	148,7066543210	-9,1891433745	291,5140765844	807,9273940741
14	10	127,8816896780	-8,4224506929	454,0609002065	825,0398828063
14	11	111,3817699729	-7,6266971302	596,6235832604	854,8238304463
14	12	97,9278246914	-6,8636059259	722,2249188477	894,7614153086
14	13	86,5363075231	-6,1355268822	832,7192524487	943,5668941976
15	8	179,0513405762	-8,5071530762	0,0000000000	813,5568182129
15	9	151,6274451540	-8,2566923172	189,3157319524	803,6572346196
15	10	130,6387362674	-7,7228786289	354,3906973785	810,9441335713
15	11	114,0457797966	-7,0973951863	499,7526628800	831,1920110198
15	12	100,5718881836	-6,4648224487	628,5859288205	861,6352648315
15	13	89,3466123861	-5,8578554309	743,0997382709	900,5262051665
15	14	79,6796956870	-5,2751393667	844,6143938637	946,9889014846
16	9	154,2518689085	-7,3527348129	92,2865976624	804,8901545650
16	10	133,0926552303	-7,0374903483	259,4703005026	803,4179489468
16	11	116,3971900144	-6,5718807983	407,1074446942	815,2259119510
16	12	102,8620227960	-6,0590262781	538,4703518878	837,4056164917
16	13	91,6475110414	-5,5485234808	655,9153003723	867,9864133589
16	14	82,1334839298	-5,0573990501	761,0897304685	905,7302132374
16	15	73,8281218530	-4,5839766095	854,9400915790	950,0229759376
17	9	156,6104758421	-6,4764602745	0,0000000000	810,4190113397
17	10	135,3069770991	-6,3698625234	168,9795641122	801,0660748802
17	11	118,4974933487	-6,0543187349	318,5208867246	805,3180627394
17	12	104,8944939376	-5,6546733211	451,9486020413	820,1513691949
17	13	93,6414079430	-5,2310447166	571,6961830632	843,4861778660
17	14	84,1578079201	-4,8133017972	679,5480456810	873,8803351313
17	15	75,9876912684	-4,4100612544	776,7517032846	910,4428918550
17	16	68,7761850391	-4,0203992390	863,9866274899	952,7308021373
18	10	137,3196901001	-5,7208401228	82,5925913725	802,8356541137
18	11	120,3965503416	-5,5477052124	233,7625216775	800,2584198483
18	12	106,7179571420	-5,2548692706	368,9237739923	808,4878348626
18	13	95,4179152353	-4,9135219393	490,5582072725	825,3579958906
18	14	85,9129822797	-4,5606570913	600,5193900565	849,3339891000
18	15	77,7846697341	-4,2145025451	700,1840825530	879,3395044075
18	16	70,6902823246	-3,8792292982	790,5080136386	914,7252389325
18	17	64,3706903919	-3,5548196830	871,9769987244	955,1618993620

NOTE α , β , μ and ϵ are all in units of 1×10^{-3} .

Table C.1 (continued)

m	n	α	β	μ	ε
19	10	139,1496250000	-5,0900181250	0,0000000000	807,9096187500
19	11	122,1302375000	-5,0534809375	152,5838471875	799,1198428125
19	12	108,3704562500	-4,8618459375	289,2318165625	801,3602625000
19	13	97,0188250000	-4,5986040625	412,4553559375	812,3967434375
19	14	87,4809000000	-4,3069634375	524,1508350000	830,6312000000
19	15	79,3443750000	-4,0105056250	625,7586734375	854,9021531250
19	16	72,2973312500	-3,7204237500	718,3571609375	884,3945350000
19	17	66,0780873071	-3,4385965290	802,6848402810	918,6300659873
19	18	60,4951234568	-3,1657522324	879,0853247478	957,3563882895
20	11	123,7207246907	-4,5719038494	74,7399526898	801,1790116264
20	12	109,8822471135	-4,4770355488	212,6836623662	797,9482811738
20	13	98,4738232381	-4,2879392332	337,2732389272	803,6777212196
20	14	88,8993849835	-4,0546864523	450,4338248217	816,7130862373
20	15	80,7401190433	-3,8047814139	553,6438253890	835,8437320329
20	16	73,6945982033	-3,5536092812	648,0414354618	860,1735387686
20	17	67,5243573136	-3,3080573368	734,4814490502	889,0784510048
20	18	62,0270511202	-3,0688692068	813,5380807649	922,2028072690
20	19	57,0593311634	-2,8372923418	885,4495276379	959,3470694381
21	11	125,1805042688	-4,1027870814	0,0000000000	805,8572211871
21	12	111,2748584476	-4,1010407267	139,0856144175	797,6054376202
21	13	99,8073278954	-3,9827324033	264,8685742314	798,4725915308
21	14	90,1927034195	-3,8051093799	379,2915229528	806,7637854827
21	15	82,0068958400	-3,5996961022	483,8588877922	821,2259618217
21	16	74,9465754505	-3,3846323534	579,7432762887	840,9212051713
21	17	68,7848146833	-3,1701150195	667,8566522885	865,1477737296
21	18	63,3357410645	-2,9603773312	748,8832650493	893,4238105389
21	19	58,4412075437	-2,7556394531	823,2713052490	925,4824714209
21	20	53,9924872844	-2,5574642897	891,1802616762	961,1609917803
22	12	112,5622493763	-3,7339426543	68,2498992309	799,8134564378
22	13	101,0383585659	-3,6836764565	195,0883064772	796,2007530338
22	14	91,3804604560	-3,5591868547	310,6161204268	800,1345064303
22	15	83,1655367136	-3,3963282816	416,3557572996	810,3242215503
22	16	76,0857406653	-3,2157585729	513,5029910799	825,8000270835
22	17	69,9154470902	-3,0297597429	603,0017042238	845,8218001615
22	18	64,4795955687	-2,8451649972	685,5911008461	869,8337436326
22	19	59,6311106506	-2,6645660073	761,8232057644	897,4613278831
22	20	55,2451821096	-2,4879744683	832,0484727783	928,5025656429
22	21	51,2381885483	-2,3171119644	896,3673255616	962,8206447076

NOTE α , β , μ and ε are all in units of 1×10^{-3} .

Table C.1 (continued)

m	n	α	β	μ	ε
23	12	113,7531148245	-3,3756614624	0,0000000000	804,1474989583
23	13	102,1805929155	-3,3910352539	127,7797799222	796,3938026565
23	14	92,4787143782	-3,3175684539	244,2868537307	796,3022860399
23	15	84,2320650394	-3,1954304107	351,0543166209	802,5583945106
23	16	77,1306716018	-3,0479567336	449,2947092156	814,1621969549
23	17	70,9462283196	-2,8891287395	539,9727159165	830,3483079467
23	18	65,5067730517	-2,7274270713	623,8577387905	850,5239901982
23	19	60,6745439037	-2,5674672567	701,5547596051	874,2452031077
23	20	56,3317476641	-2,4108249757	773,5119026250	901,2192903703
23	21	52,3789712444	-2,2574588071	840,0031107829	931,2919267259
23	22	48,7509673306	-2,1091382204	901,0843478372	964,3448710375
24	13	103,2433478819	-3,1048361929	62,7962963934	798,6676773352
24	14	93,4998991613	-3,0805979769	180,1796657031	794,8416535458
24	15	85,2198224000	-2,9975602432	287,8595445984	797,4563606400
24	16	78,0948480411	-2,8818367882	387,0601172274	805,4868099248
24	17	71,8941228860	-2,7491283441	478,7611920828	818,1445544127
24	18	66,4447043048	-2,6091448018	563,7511952496	834,8153636945
24	19	61,6126915633	-2,4678299408	642,6640805184	855,0189514195
24	20	57,2879088000	-2,3283191552	715,9989838080	878,3981891840
24	21	53,3750541943	-2,1915606657	784,1214493452	904,7233952180
24	22	49,7942298597	-2,0575307047	847,2450550466	933,8754408471
24	23	46,4937670005	-1,9279731656	905,3922645488	965,7495722974
25	13	104,2328856132	-2,8250501511	0,0000000000	802,7013015441
25	14	94,4531438920	-2,8483968323	118,1726878830	795,4024387937
25	15	86,1396570015	-2,8030714582	226,6706783537	794,6305407379
25	16	78,9893820242	-2,7178609400	326,7248166681	799,3466482243
25	17	72,7708231743	-2,6102752025	419,3261414998	808,7406844958
25	18	67,3090611675	-2,4911808278	505,2766660081	822,1806443986
25	19	62,4704476832	-2,3674613613	585,2280935412	839,1665981029
25	20	58,1487801571	-2,2433309433	659,7075915449	859,3057874761
25	21	54,2547721412	-2,1209279325	729,1297472481	882,3094990236
25	22	50,7106344695	-2,0008151849	793,7938286936	907,9968030989
25	23	47,4515824664	-1,8830136520	853,8654746859	936,2746548624
25	24	44,4360844355	-1,7691959649	909,3419372255	967,0482582508
26	14	95,3449516529	-2,6209763465	58,1493461856	797,6921057014
26	15	87,0000110601	-2,6121400166	167,3864953313	793,7600455859
26	16	79,8235563470	-2,5563562391	268,2070524346	795,3879797465
26	17	73,5857124933	-2,4729505699	361,6097876691	801,7490889242
26	18	68,1102550823	-2,3739926858	448,4070425448	812,1940558767
26	19	63,2623426172	-2,2671844225	529,2628483474	826,2086884902
26	20	58,9367927789	-2,1574866751	604,7212241224	843,3813295696
26	21	55,0480429364	-2,0479145245	675,2239918901	863,3884840036
26	22	51,5229352216	-1,9399299519	741,1174467776	885,9958055677
26	23	48,2974664808	-1,8338615040	802,6472197534	911,0602971929
26	24	45,3186434134	-1,7297802734	859,9406706514	938,5082900934
26	25	42,5525832097	-1,6292615541	912,9761491712	968,2524787108

NOTE α , β , μ and ε are all in units of 1×10^{-3} .

Table C.1 (continued)

m	n	α	β	μ	ϵ
27	14	96,1799524157	-2,3983307950	0,0000000000	801,4620973787
27	15	87,8072339510	-2,4248248792	109,9084470023	794,5762402919
27	16	80,6049085854	-2,3975186913	211,4227629241	793,3158799216
27	17	74,3466955590	-2,3374412078	305,5459346098	796,8463988729
27	18	68,8563635730	-2,2578996096	393,0980364707	804,5028770565
27	19	63,9978278861	-2,1674187844	474,7524530084	815,7576173705
27	20	59,6654080049	-2,0715671753	551,0645680133	830,1867939907
27	21	55,7749666285	-1,9739678436	622,4924148013	847,4482192120
27	22	52,2566505091	-1,8767936096	689,4087818496	867,2739476397
27	23	49,0499538933	-1,7810451416	752,1042681971	889,4731593849
27	24	46,1018252034	-1,6869108574	810,7807829691	913,9324867793
27	25	43,3685376227	-1,5945075052	865,5349833899	940,5926719763
27	26	40,8220442466	-1,5053002920	916,3311456462	969,3721656679
28	15	88,5660259125	-2,2411328182	54,1424788011	796,8511527567
28	16	81,3393701057	-2,2414413970	156,2886460959	792,8824325275
28	17	75,0603739817	-2,2039406070	251,0647971307	793,7610598373
28	18	69,5541501973	-2,1431471452	339,2962419039	798,8109866022
28	19	64,6839991218	-2,0684387560	421,6636413417	807,4913705645
28	20	60,3433137634	-1,9859661249	498,7308586043	819,3677667693
28	21	56,4479788544	-1,8998226295	570,9665830432	834,0869174986
28	22	52,9297209889	-1,8126830339	638,7593370987	851,3622108001
28	23	49,7308667081	-1,7261222115	702,4254764256	870,9707362940
28	24	46,8009654263	-1,6408249821	762,2097935366	892,7567254884
28	25	44,0957340937	-1,5568981499	818,2783352524	916,6300223837
28	26	41,5787804887	-1,4744958266	870,7030442470	942,5420886878
28	27	39,2265620349	-1,3949691273	919,4378349773	970,4159065169
29	15	89,2798839506	-2,0610683539	0,0000000000	800,3884370370
29	16	82,0315098349	-2,0881547819	102,7240365303	793,8770365482
29	17	75,7321296520	-2,0725597540	198,0959608027	792,2635822278
29	18	70,2093506173	-2,0299142798	286,9437737860	794,8680140741
29	19	65,3267056512	-1,9704535803	369,9535738121	801,1382490326
29	20	60,9768578172	-1,9009261993	447,6958995522	810,6292670577
29	21	57,0751222222	-1,8258524691	520,6472033745	822,9791690123
29	22	53,5535949791	-1,7482834402	589,2061520394	837,8907417215
29	23	50,3561788346	-1,6702113813	653,7044516931	855,1224584510
29	24	47,4347950617	-1,5927829630	714,4118941152	874,4882360494
29	25	44,7470712190	-1,5164662295	771,5353211803	895,8606629483
29	26	42,2557943767	-1,4413224717	825,2112044901	919,1678051803
29	27	39,9304194120	-1,3675341081	875,4915371353	944,3690905327
29	28	37,7509219731	-1,2963396833	922,3227345445	971,3911639175

NOTE α , β , μ and ϵ are all in units of 1×10^{-3} .

Table C.1 (continued)

m	n	α	β	μ	ε
30	16	82,6848208854	-1,9376644008	50,6520145730	796,1185722795
30	17	76,3662564658	-1,9433496245	146,5702475018	792,1584763197
30	18	70,8267629538	-1,9183170174	235,9811143595	792,4611315875
30	19	65,9309300986	-1,8736230328	319,5743135792	796,4671299289
30	20	61,5711678648	-1,8166270961	397,9256281462	803,7214545789
30	21	57,6621596530	-1,7522778385	471,5175920251	813,8538227737
30	22	54,1357421601	-1,6839506825	540,7560773794	826,5596155283
30	23	50,9363946094	-1,6139463118	605,9825649718	841,5868189642
30	24	48,0175200857	-1,5437595603	667,4818601282	858,7309102420
30	25	45,3387017070	-1,4742282493	725,4850166532	877,8361298288
30	26	42,8641163652	-1,4056715102	780,1672310812	898,7980905027
30	27	40,5622887708	-1,3381271218	831,6404696499	921,5591821698
30	28	38,4073685313	-1,2717973971	879,9405903814	946,0847402411
30	29	36,3821129993	-1,2078131518	925,0087226562	972,3044539924
31	16	83,3019925385	-1,7899787388	0,0000000000	799,4492403168
31	17	76,9661360877	-1,8163270487	96,4208836722	793,2775680743
31	18	71,4103278105	-1,8084196718	186,3489334956	791,4083191149
31	19	66,5009117575	-1,7780597559	270,4755669886	793,2803915678
31	20	62,1305948377	-1,7332089792	349,3804542382	798,4306924738
31	21	58,2136500316	-1,6792554773	423,5510178697	806,4805828440
31	22	54,6814518197	-1,6198892883	493,3986895403	817,1187339274
31	23	51,4784579361	-1,5576656387	559,2717351874	830,0864157799
31	24	48,5587515564	-1,4943363914	621,4644612582	845,1685682487
31	25	45,8832580326	-1,4310299785	680,2226141518	862,1913335039
31	26	43,4177502831	-1,3683601397	735,7447851016	881,0240582666
31	27	41,1317569496	-1,3065437907	788,1796327268	901,5811029053
31	28	38,9984874307	-1,2456083418	837,6187354827	923,8161235884
31	29	36,9958879027	-1,1857687888	884,0848862383	947,6988227000
31	30	35,1089424384	-1,1280548995	927,5156412107	973,1614917466

NOTE α , β , μ and ε are all in units of 1×10^{-3} .

Table C.2 – Fractiles of the F-distribution, $F(0,95 f_n f_d)$

f f_d	f_n								
	1	2	3	4	5	6	7	8	9
10	4,965	4,103	3,708	3,478	3,326	3,217	3,135	3,072	3,02
11	4,844	3,982	3,587	3,357	3,204	3,095	3,012	2,948	2,896
12	4,747	3,885	3,49	3,259	3,106	2,996	2,913	2,849	2,796
13	4,667	3,806	3,411	3,179	3,025	2,915	2,832	2,767	2,714
14	4,6	3,739	3,344	3,112	2,958	2,848	2,764	2,699	2,646
15	4,543	3,682	3,287	3,056	2,901	2,79	2,707	2,641	2,588
16	4,494	3,634	3,239	3,007	2,852	2,741	2,657	2,591	2,538
17	4,451	3,592	3,197	2,965	2,81	2,699	2,614	2,548	2,494
18	4,414	3,555	3,16	2,928	2,773	2,661	2,577	2,51	2,456
19	4,381	3,522	3,127	2,895	2,74	2,628	2,544	2,477	2,423
20	4,351	3,493	3,098	2,866	2,711	2,599	2,514	2,447	2,393
25	4,242	3,385	2,991	2,759	2,603	2,49	2,405	2,337	2,282
30	4,171	3,316	2,922	2,69	2,534	2,421	2,334	2,266	2,211
40	4,085	3,232	2,839	2,606	2,449	2,336	2,249	2,18	2,124
50	4,034	3,183	2,79	2,557	2,4	2,286	2,199	2,13	2,073
100	3,936	3,087	2,696	2,463	2,305	2,191	2,103	2,032	1,975
500	3,86	3,014	2,623	2,39	2,232	2,117	2,028	1,957	1,899

f f_d	f_n								
	10	11	12	13	14	15	16	17	18
10	2,978	2,943	2,913	2,887	2,865	2,845	2,828	2,812	2,798
11	2,854	2,818	2,788	2,761	2,739	2,719	2,701	2,685	2,671
12	2,753	2,717	2,687	2,66	2,637	2,617	2,599	2,583	2,568
13	2,671	2,635	2,604	2,577	2,554	2,533	2,515	2,499	2,484
14	2,602	2,565	2,534	2,507	2,484	2,463	2,445	2,428	2,413
15	2,544	2,507	2,475	2,448	2,424	2,403	2,385	2,368	2,353
16	2,494	2,456	2,425	2,397	2,373	2,352	2,333	2,317	2,302
17	2,45	2,413	2,381	2,353	2,329	2,308	2,289	2,272	2,257
18	2,412	2,374	2,342	2,314	2,29	2,269	2,25	2,233	2,217
19	2,378	2,34	2,308	2,28	2,256	2,234	2,215	2,198	2,182
20	2,348	2,31	2,278	2,25	2,225	2,203	2,184	2,167	2,151
25	2,236	2,198	2,165	2,136	2,111	2,089	2,069	2,051	2,035
30	2,165	2,126	2,092	2,063	2,037	2,015	1,995	1,976	1,96
40	2,077	2,038	2,003	1,974	1,948	1,924	1,904	1,885	1,868
50	2,026	1,986	1,952	1,921	1,895	1,871	1,85	1,831	1,814
100	1,927	1,886	1,85	1,819	1,792	1,768	1,746	1,726	1,708
500	1,85	1,808	1,772	1,74	1,712	1,686	1,664	1,643	1,625

	19	20	25	30	40	50	100	500
10	2,785	2,774	2,73	2,7	2,661	2,637	2,588	2,548
11	2,658	2,646	2,601	2,57	2,531	2,507	2,457	2,415
12	2,555	2,544	2,498	2,466	2,426	2,401	2,35	2,307
13	2,471	2,459	2,412	2,38	2,339	2,314	2,261	2,218
14	2,4	2,388	2,341	2,308	2,266	2,241	2,187	2,142
15	2,34	2,328	2,28	2,247	2,204	2,178	2,123	2,078
16	2,288	2,276	2,227	2,194	2,151	2,124	2,068	2,022
17	2,243	2,23	2,181	2,148	2,104	2,077	2,02	1,973
18	2,203	2,191	2,141	2,107	2,063	2,035	1,978	1,929
19	2,168	2,155	2,106	2,071	2,026	1,999	1,94	1,891
20	2,137	2,124	2,074	2,039	1,994	1,966	1,907	1,856
25	2,021	2,007	1,955	1,919	1,872	1,842	1,779	1,725
30	1,945	1,932	1,878	1,841	1,792	1,761	1,695	1,637
40	1,853	1,839	1,783	1,744	1,693	1,66	1,589	1,526
50	1,798	1,784	1,727	1,687	1,634	1,599	1,525	1,457
100	1,691	1,676	1,616	1,573	1,515	1,477	1,392	1,308
500	1,607	1,592	1,528	1,482	1,419	1,376	1,275	1,159

Table C.3 – Fractiles of the F -distribution, $F(0,995, f_n, f_d)$

f f_d	f_n								
	1	2	3	4	5	6	7	8	9
10	12,826	9,427	8,081	7,343	6,872	6,545	6,302	6,116	5,968
11	12,226	8,912	7,6	6,881	6,422	6,102	5,865	5,682	5,537
12	11,754	8,51	7,226	6,521	6,071	5,757	5,525	5,345	5,202
13	11,374	8,186	6,926	6,233	5,791	5,482	5,253	5,076	4,935
14	11,06	7,922	6,68	5,998	5,562	5,257	5,031	4,857	4,717
15	10,798	7,701	6,476	5,803	5,372	5,071	4,847	4,674	4,536
16	10,575	7,514	6,303	5,638	5,212	4,913	4,692	4,521	4,384
17	10,384	7,354	6,156	5,497	5,075	4,779	4,559	4,389	4,254
18	10,218	7,215	6,028	5,375	4,956	4,663	4,445	4,276	4,141
19	10,073	7,093	5,916	5,268	4,853	4,561	4,345	4,177	4,043
20	9,944	6,986	5,818	5,174	4,762	4,472	4,257	4,09	3,956
25	9,475	6,598	5,462	4,835	4,433	4,15	3,939	3,776	3,645
30	9,18	6,355	5,239	4,623	4,228	3,949	3,742	3,58	3,45
40	8,828	6,066	4,976	4,374	3,986	3,713	3,509	3,35	3,222
50	8,626	5,902	4,826	4,232	3,849	3,579	3,376	3,219	3,092
100	8,241	5,589	4,542	3,963	3,589	3,325	3,127	2,972	2,847
500	7,95	5,355	4,33	3,763	3,396	3,137	2,941	2,789	2,665

f f_d	f_n								
	10	11	12	13	14	15	16	17	18
10	5,847	5,746	5,661	5,589	5,526	5,471	5,422	5,379	5,34
11	5,418	5,32	5,236	5,165	5,103	5,049	5,001	4,959	4,921
12	5,085	4,988	4,906	4,836	4,775	4,721	4,674	4,632	4,595
13	4,82	4,724	4,643	4,573	4,513	4,46	4,413	4,372	4,334
14	4,603	4,508	4,428	4,359	4,299	4,247	4,2	4,159	4,122
15	4,424	4,329	4,25	4,181	4,122	4,07	4,024	3,983	3,946
16	4,272	4,179	4,099	4,031	3,972	3,92	3,875	3,834	3,797
17	4,142	4,05	3,971	3,903	3,844	3,793	3,747	3,707	3,67
18	4,03	3,938	3,86	3,793	3,734	3,683	3,637	3,597	3,56
19	3,933	3,841	3,763	3,696	3,638	3,587	3,541	3,501	3,465
20	3,847	3,756	3,678	3,611	3,553	3,502	3,457	3,416	3,38
25	3,537	3,447	3,37	3,304	3,247	3,196	3,151	3,111	3,075
30	3,344	3,255	3,179	3,113	3,056	3,006	2,961	2,921	2,885
40	3,117	3,028	2,953	2,888	2,831	2,781	2,737	2,697	2,661
50	2,988	2,9	2,825	2,76	2,703	2,653	2,609	2,569	2,533
100	2,744	2,657	2,583	2,518	2,461	2,411	2,367	2,326	2,29
500	2,562	2,476	2,402	2,337	2,281	2,23	2,185	2,145	2,108

f f_d	f_n							
	19	20	25	30	40	50	100	500
10	5,305	5,274	5,153	5,071	4,966	4,902	4,772	4,666
11	4,886	4,855	4,736	4,654	4,551	4,488	4,359	4,252
12	4,561	4,53	4,412	4,331	4,228	4,165	4,037	3,931
13	4,301	4,27	4,153	4,073	3,97	3,908	3,78	3,674
14	4,089	4,059	3,942	3,862	3,76	3,698	3,569	3,463
15	3,913	3,883	3,766	3,687	3,585	3,523	3,394	3,287
16	3,764	3,734	3,618	3,539	3,437	3,375	3,246	3,139
17	3,637	3,607	3,492	3,412	3,311	3,248	3,119	3,012
18	3,527	3,498	3,382	3,303	3,201	3,139	3,009	2,901
19	3,432	3,402	3,287	3,208	3,106	3,043	2,913	2,804
20	3,347	3,318	3,203	3,123	3,022	2,959	2,828	2,719
25	3,043	3,013	2,898	2,819	2,716	2,652	2,519	2,406
30	2,853	2,823	2,708	2,628	2,524	2,459	2,323	2,207
40	2,628	2,598	2,482	2,401	2,296	2,23	2,088	1,965
50	2,5	2,47	2,353	2,272	2,164	2,097	1,951	1,821
100	2,257	2,227	2,108	2,024	1,912	1,84	1,681	1,529
500	2,075	2,044	1,922	1,835	1,717	1,64	1,46	1,26

Table C.4 –Fractiles of the t -distribution, $t_{0,95}$

f	t
1	6,314
2	2,920
3	2,353
4	2,132
5	2,015
6	1,943
7	1,895
8	1,860
9	1,833
10	1,812
11	1,796
12	1,782
13	1,771
14	1,761
15	1,753
16	1,746
17	1,740
18	1,734
19	1,729
20	1,725
25	1,708
30	1,697
40	1,684
50	1,676
100	1,660
500	1,64

Table C.5 – Fractiles of the χ^2 -distribution

f	$p = 0,95$	$p = 0,99$	$p = 0,995$
1	3,8	6,6	7,9
2	6,0	9,2	10,6
3	7,8	11,3	12,8
4	9,5	13,3	14,9
5	11,1	15,1	16,7
6	12,6	16,8	18,5

NOTE The significance level P is equal to $(1-p)$, for example significance 0,05 corresponds to $p = 0,95$.

Annex D (informative)

Worked examples

Table D.1 – Worked example 1 – Censored data (proof tests)

ϑ_i	240		260		280	
x_i	0,001948747929		0,001875644753		0,001807827895	
j	r_{ij}	y_{ij}	r_{ij}	y_{ij}	r_{ij}	y_{ij}
1	1764	7,475339237	756	6,628041376	108	4,682131227
2	2772	7,927324360	924	6,828712072	252	5,529429088
3	2772	7,927324360	924	6,828712072	324	5,780743516
4	3780	8,237479289	1176	7,069874128	324	5,780743516
5	4284	8,362642432	1176	7,069874128	468	6,148468296
6	4284	8,362642432	2184	7,688913337	612	6,416732283
7	4284	8,362642432	2520	7,832014181	684	6,527957918
8	5292	8,573951525	2856	7,957177323	756	6,628041376
9	7308	8,896724917	2856	7,957177323	756	6,628041376
10	7812	8,963416292	3192	8,068402959	828	6,719013154
11	7812	8,963416292	3192	8,068402959	828	6,719013154
12			3864	8,259458195	972	6,879355804
13			4872	8,491259809	1428	7,264030143
14			5208	8,557951184	1596	7,375255778
15			5544	8,620471541	1932	7,566311015
16			5880	8,679312041	1932	7,566311015
17			5880	8,679312041	2100	7,649692624
18			5880	8,679312041	2268	7,726653665
19					2604	7,864804003
20					2772	7,927324360
m_i	21		21		21	
n_i	11		18		20	
α_i	0,12518050427		0,06333574106		0,05399248728	
β_i	-0,00410278708		-0,00296037733		-0,00255746429	
μ_i	0		0,74888326505		0,89118026168	
ε_i	0,80585722119		0,89342381054		0,96116099178	
$\sum_{j=1}^{n_i-1} y_{ij}$	83,0894872752		133,285066669		127,45272895	
$\sum_{j=1}^{n_i-1} (y_{in_i} - y_{ij})^2$	6,12724907570		19,9557443468		41,4224423138	
\bar{y}_i	8,963416292		8,050988496		6,84072074866	
$s_{y_i}^2$	0,59127835553		0,66165281385		0,863951396023	

Table D.1 (continued)

$\sum_{i=1}^k \varepsilon_i / k$	0,886814007835	(29)
$\sum_{i=1}^k n_i x_i^2$	0,000170463415664	
$\sum_{i=1}^k n_i \bar{y}_i^2$	2986,41183881	
$\sum_{i=1}^k n_i x_i \bar{y}_i$	0,711293042041	
$M = \sum_{i=1}^k m_i$	63	(28)
$N = \sum_{i=1}^k n_i$	49	(25)
$\sum_{i=1}^k n_i x_i / N$	0,00186437531983	(26)
$\sum_{i=1}^k n_i \bar{y}_i / N$	7,76183239007	(27)
b	15327,98578	(33)
a	-20,8152860044	(34)
s_1^2	0,647296300122	(30)
s_2^2	0,395498398826	(36)
F	0,611000555311	(40)
χ^2	0,554692947413	(38)
c	1,03161932965	(39)
$t_{0,95, N-2}$	1,677926722	(43)
t_c	1,73895334031	(43)
$\mu_2(x)$	$2,9498844403 \times 10^{-9}$	(31)
s^2	0,641938897967	(41)
$TI = \hat{\vartheta}$	225,827791333	(50)
$TC = \hat{\vartheta}_c$	214,550619764	(50)
HIC	11,5189953038	(53)
$(TI - TC)/HIC$	0,979006525432	
TI_a	221,462017221	(55)
Result	TI(HIC):221,5(11,5)	

**Table D.2 – Worked example 2 – Complete data
(non-destructive tests)**

θ_i	180		200		220	
x_i	0,002206774799		0,002113494663		0,002027780594	
j	τ_{ij}	y_{ij}	τ_{ij}	y_{ij}	τ_{ij}	y_{ij}
1	7410	8,910585718	3200	8,070906089	1100	7,003065459
2	6610	8,796338933	2620	7,870929597	740	6,606650186
3	6170	8,727454117	2460	7,807916629	720	6,579251212
4	5500	8,612503371	2540	7,839919360	620	6,429719478
5	8910	9,094929520	3500	8,160518247	910	6,813444599

m_i	5	5	5
n_i	5	5	5
ε_i	1	1	1
$\sum_{j=1}^{n_i} y_{ij}$	44,14181166	39,75018992	33,43213093
$\sum_{j=1}^{n_i} y_{ij}^2$	389,8355291	316,1130135	223,741618
\bar{y}_i	8,828362332	7,950037984	6,686426187
$s_{\tau_i}^2$	0,03390545203	0,0024373442	0,00500357814

Table D.2 (continued)

$\sum_{i=1}^k \varepsilon_i / k$	1	(29)
$\sum_{i=1}^k n_i x_i^2$	$6,7243044211 \times 10^{-5}$	
$\sum_{i=1}^k n_i \bar{y}_i^2$	929,25690285	
$\sum_{i=1}^k n_i x_i \bar{y}_i$	0,24921587814	
$M = \sum_{i=1}^k m_i$	15	(28)
$N = \sum_{i=1}^k n_i$	15	(25)
$\sum_{i=1}^k n_i x_i / N$	$2,1160166854 \times 10^{-3}$	(26)
$\sum_{i=1}^k n_i \bar{y}_i / N$	7,8216088344	(27)
b	11929,077582	(33)
a	-17,42051837	(34)
s_1^2	0,0361048918	(30)
s_2^2	0,18856369729	(36)
F	5,222663409	(40)
F_0	4,747	
χ^2	0,466116435248	(38)
c	1,1111111111	(39)
$t_{0,95, N-2}$	1,7709333962	(43)
t_c	1,7709333962	(43)
$\mu_2(x)$	$5,3430011710 \times 10^{-9}$	(31)
s_a^2	0,05119958608	(42)
$TI = \hat{\vartheta}$	163,428648665	(50)
$TC_a = \hat{\vartheta}_c$	158,670330155	(50)
HIC	11,3632557756	(53)
$(TI - TC_a)/HIC$	0,41874605344	
Result	TI(HIC):163(11,4)	(54)

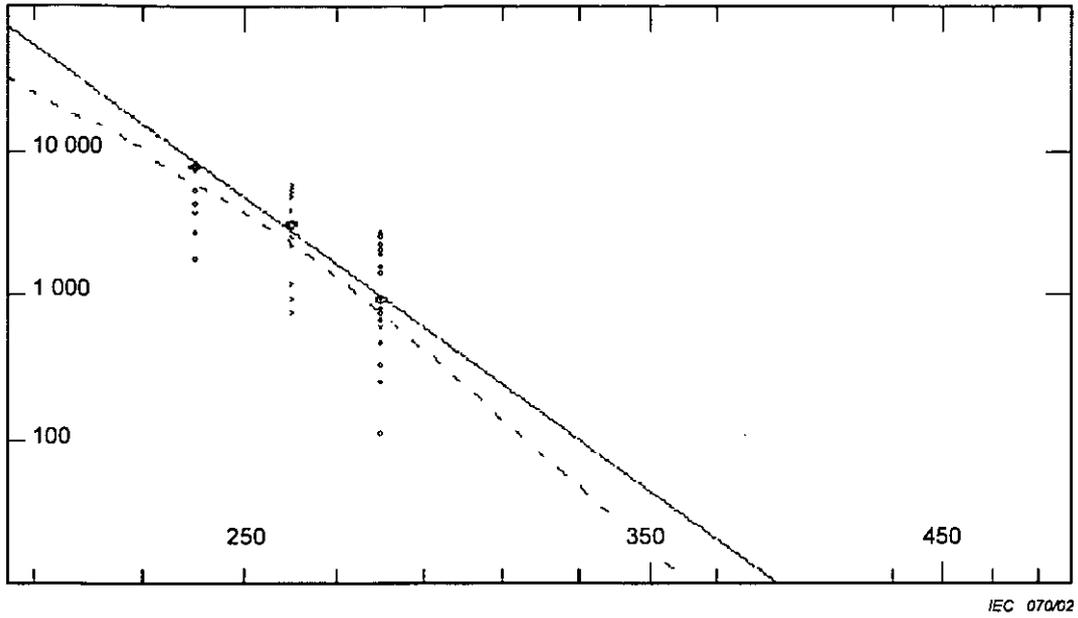


Figure D.1a - Example 1

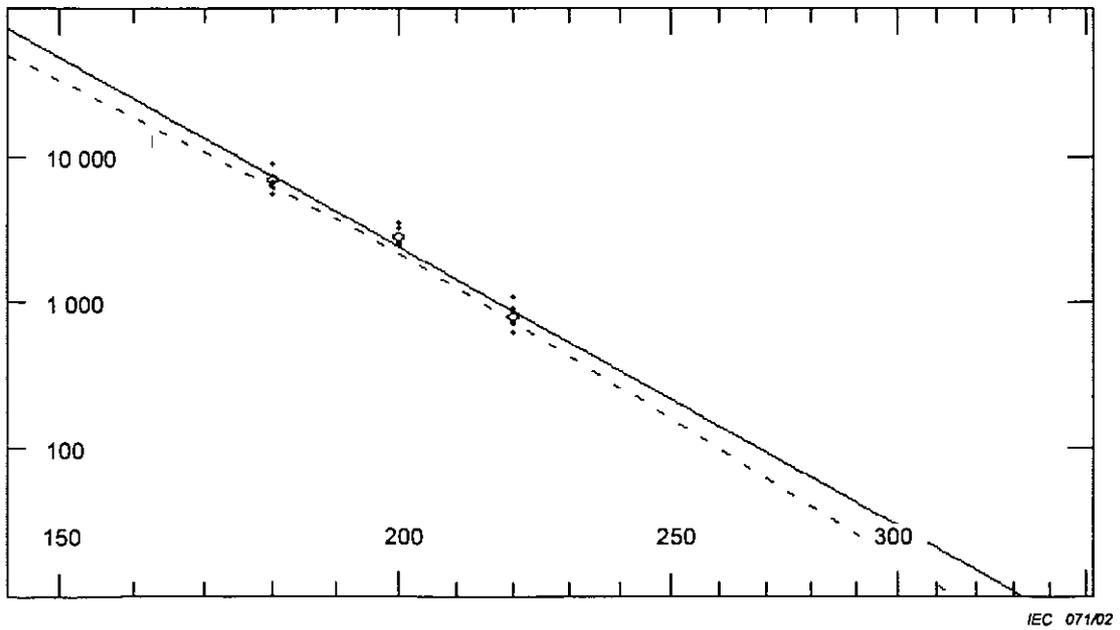


Figure D.1b - Example 2

NOTE In the above figures, the full line represents the regression equation, and the dotted line the lower 95 % confidence limit of a temperature estimate. The figures are as drawn by the programme of Annex E.

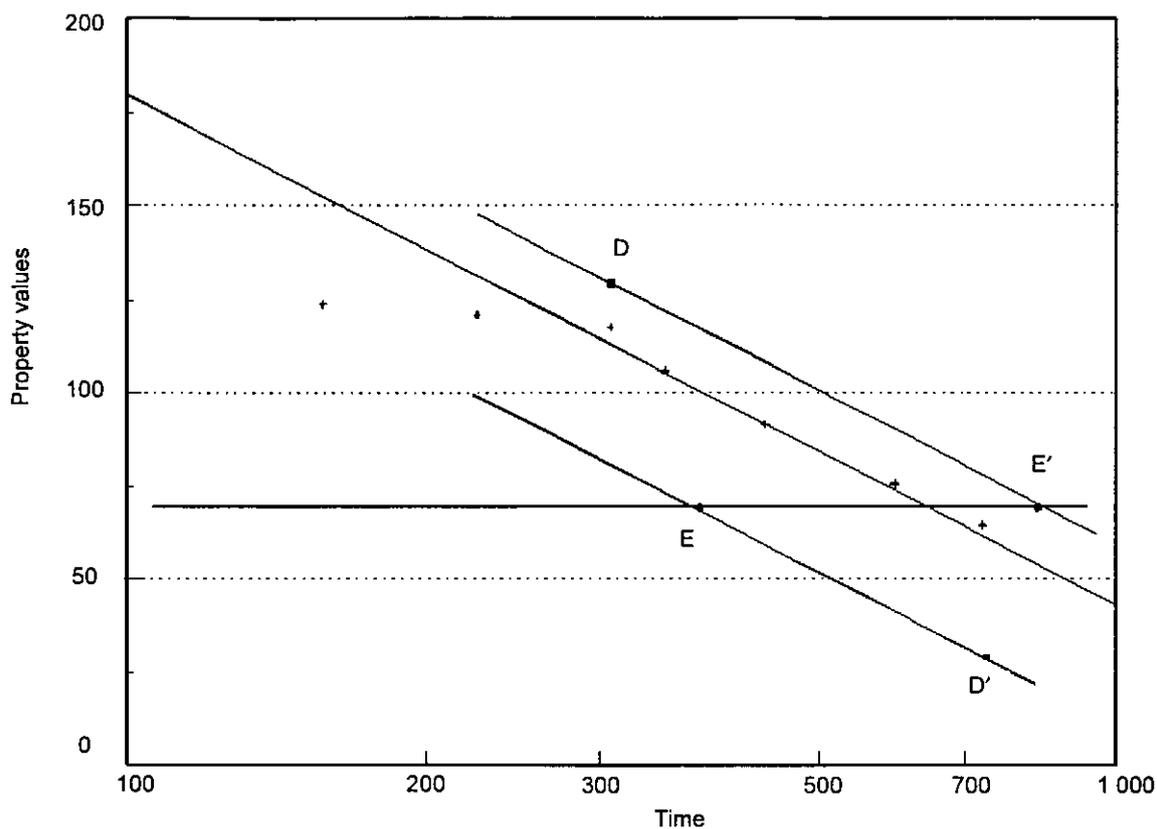
Figure D.1 - Thermal endurance graph

Table D.3 – Worked example 3 – Destructive tests

This worked example is given to illustrate the calculations specific for destructive test data, and relates to a single test temperature. The data from this calculation and further test temperatures would be entered into a calculation similar to that exemplified in worked example 2 (Table D.2).

End-point 70,0					
τ_g	288	336	432	624	720
p_{gh}	139,5	121,9	101,2	77,8	69,6
	125	109,3	99,5	74,6	69,4
	120,8	98,3	98,4	71,4	67,2
	112,7	96,5	92,4	68,2	60,4
	112	93	78,1	60,5	59,4
n_g	5	5	5	5	5
\bar{p}_g	122,00	103,80	93,92	70,50	65,20
s_{1g}^2	125,795	139,510	89,197	44,050	24,420
$\log \tau_g$	5,66296	5,817111	6,068426	6,43615	6,579251
n_i			25		
\bar{z}			6,1128		
\bar{p}			91,084		
b_p			-59,4937		
a_p			454,756		
s_1^2			84,594		
s_2^2			77,266		
F			0,913		
F_1			3,098		
z_{gh}	6,831151	6,689472	6,592851	6,567257	6,572528
	6,587428	6,477685	6,564276	6,513469	6,569166
	6,516832	6,292792	6,545787	6,459682	6,532187
	6,380683	6,262536	6,444936	6,405895	6,41789
	6,368917	6,203707	6,204574	6,27647	6,401081

In the graph below, displaying the data of example 3, the line passing through the points marked E and E' indicates the chosen end-point. The points marked D and D' are two randomly selected data points, with lines parallel to the regression line intersecting the end-point line at E and E'. The other points marked on the graph are the means of the property value groups.



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**Figure D.2 – Example 3: Property-time graph
(destructive test data)**

Extrapolation

If, in the above data set, only the data up to ageing time 624 h were available, the ageing curve would not have crossed the end-point line, since $70,5 > 70,0$. In this case, the extrapolation required would be

$$(70,5 - 70,0) / (122,0 - 70,5) = 0,0097$$

This would be permitted, subject to the other restrictions of 6.1.4.4.

Annex E (informative)

Computer program

E.1 General

The programs on the accompanying CD-ROM for making calculations in this standard are for use in conjunction with the disk operating system MSDOS or equivalent. The preferred mode of operation is in a Windows 95 (or later) system in a "DOS" window.

CD-ROM content

Annex E.doc
Entry-3.bas, Entry-3.exe
216-3.bas, 216-3.exe
Test2.dta
Cenex3.dta
N3.dst

The text based program files (*.bas) are written in a dialect of the Basic language known as "Quick Basic", chosen mainly for historical reasons. They may be edited if necessary using either a text editor or the "Quick Basic" programs QB45 or QBX and saved as ASCII files with the filename ending in ".bas". They may then be executed using QB45 or QBX or compiled for stand-alone execution. The resultant executable files will have the ending ".exe" and the same file stem as the ".bas" files.

These executable files are suitable for direct execution in DOS, and may be executed in Windows 95 or later (98 etc.) by creating an icon for the file and double-clicking that icon. The "short-cut" options should be set so that the program is run in a normal (full screen) window.

To create the icon, in Windows Explorer, right click the file name and select "Create Shortcut", or drag the program into the desktop.

NOTE It is not possible, without substantial modification, to run the text based programs in a "Visual Basic" environment.

The program code is in two separate files, one being the actual calculation processor and the other for entry of the data to a file in a format suitable for retrieval and processing by the first. The actual format or structure of these files is described in the next section, and the actual content of the files used in the worked examples is given in the subsequent section.

E.1.1 216-3.bas (or .exe)

The program requires data to be entered in the form of an ASCII text file obtained using the associated program Entry-3.bas described below.

Data analysis and statistical tests in accordance with this standard are carried out and the results reported in the appropriate format. The report is recorded in a text file with the ending ".rep", which may be edited in a word processor program.

Result parameters as required for calculation of RTE are automatically entered into a file having the same name as the experimental data file and with the ending ".int".

E.1.2 Entry-3.bas (or .exe)

Data obtained using the procedures set out in this standard are entered following the screen instructions to generate a data file. The file endings ".dta" and ".dst" are used by preference for non-destructives or proof test data and destructive test data respectively.

The data in the enclosed files may be printed and used to gain familiarity with the data entry program.

The statistical tests used in the calculations (F and t) are made with values of the statistical functions obtained from very simple approximate algorithms. They may be in error by 1 % or 2 %. This accuracy may be substantially improved by the use of accurate algorithms, but only at the expense of several further pages of computer code. Useful routines (in FORTRAN, Pascal or C) will be found in reference [2] (chapter 6 is relevant here), the FORTRAN routines having been found extremely easy to adapt.

To enable checking of the computer code to be carried out easily, three data files are provided in tabular form. These should be entered using a text editor, one number per line, with a carriage return (Enter) at the end of each line, with no blank lines. The first two data sets are those for the worked examples (1) and (2). The third (N3.dst) is for a set of destructive test data. In the latter, the data selected as the linear region is indicated in the specimen report provided.

E.2 Structure of data files used by the program

Please read Table E.1 in conjunction with the sub-routine NDEntry in Entry.bas and the list of symbols in 3.2.

The file comprises a series of numbers, with one value only on each line of the file.

Table E.1 – Non-destructive test data

Line	Item	Symbol
1	Number of temperatures	k
2	Maximum number of specimens at any temperature	
3	First ageing temperature	ϑ_1
4	Number of specimens at ϑ_1	m_1
5	Number of known times to endpoint at ϑ_1	n_1
6 to 5+ n_1	Times to endpoint at ϑ_1	t_{ij}
6+ n_1	Second ageing temperature	ϑ_2
	Number of specimens aged at ϑ_2	m_2
	Number of times known at ϑ_2	n_2
	n_2 lines containing times to endpoint	
	Third ageing temperature, etc.	

Please read Table E.2 in conjunction with sub-routine DestEntry in Entry.bas and the list of symbols in 3.2.

Table E.2 – Destructive test data

Line	Item	Symbol
1	Number of ageing temperatures	k
2	Largest number of ageing times at any temperature	
3	Largest number of specimens aged in any group	
4	First ageing temperature	ϑ_1
5	Number of groups aged at ϑ_1	
6	Ageing time for first group at ϑ_1	
7	Number of specimens aged in this group	
8 and subsequently	Property values for specimens in this group	
	Ageing time for next group Number of specimens aged in this group Property values for specimens in this group	
	Ageing time for next group Number of specimens aged in this group Property values for specimens in this group Etc.	
	Second ageing temperature	ϑ_2
	Number of groups aged at ϑ_2 Ageing time for first group at ϑ_2 Number of specimens aged in this group Property values for specimens in this group	
	Ageing time for next group Number of specimens aged in this group Property values for specimens in this group Etc.	
	Third ageing temperature, etc.	ϑ_3

E.3 Data files for computer program

The following pages show the file structure for the data of Examples 1 and 2, and a complete data file for a destructive test (designated Material N3). The calculated results are also given.

The data files are in the format prepared by the program Entry.bas above, but it may also be prepared using a text editor.

Material: cenex3 sleeving

File name: ex-1.dta Estimate time: 20 000 02-27-1995

Test property: voltage proof test

Data dispersion slightly too large, compensated

TI (HIC) : 221,5 (11.5) TC 214,6

Chi-squared = 0,56 (2 DF)

$F = 0,610 : F(0,95, 1, 46) = 4,099$

Times to reach end-point

Temperature 240

Number of specimens 21, times known for 11

Times 1764 2772 2772 3780 4284 4284 4284 5292 7308 7812 7812

Temperature 260

Number of specimens 21, times known for 18

Times 756 924 924 1176 1176 2184 2520 2856 2856 3192 3192 3864
4872 5208 5544 5880 5880 5880

Temperature 280

Number of specimens 21, times known for 20

Times 108 252 324 324 468 612 684 756 756 828 828 972 1428
1596 1932 1932 2100 2268 2604 2772

Data file Cenex3.dta (Example 1)

Data at the foot of each column are followed without interruption by those in the succeeding column.

3	924	324
21	1176	324
240	1176	468
21	2184	612
11	2520	684
1764	2856	756
2772	2856	756
2772	3192	828
3780	3192	828
4284	3864	972
4284	4872	1428
4284	5208	1596
5292	5544	1932
7308	5880	1932
7812	5880	2100
7812	5880	2268
260	280	2604
21	21	2772
18	20	
756	108	
924	252	

Material: Unidentified resin

File name: test2 Estimate time: 20 000 12-02-1991

Test property: Loss of mass

Minor non-linearity, compensated
TI (HIC) : 163,4 (11,4) TC 158,7

Chi-squared = 0,48 (2 DF)
 $F = 5,223 : F(0,95, 1, 12) = 4,743$

Times to reach end-point

Temperature 180

Times 7410 6610 6170 5500 8910

Temperature 200

Times 3200 2620 2460 2540 3500

Temperature 220

Times 1100 740 720 620 910

Data file test2.dta (example 2)

3		
5		
180	200	220
5	5	5
5	5	5
7410	3200	1100
6610	2620	740
6170	2460	720
5500	2540	620
8910	3500	910

Material: N3 nylon laminate

File name: n3.dst Estimate time: 20 000 12-02-1991

Test property: Tensile impact strength (end-point 30)

TI (HIC) : 113,8 (12,4) TC 112,4

Chi-squared = 42,63 (3 DF)
F = 1,772 : F(0,95, 2, 101) = 2,975

Temperature 180 Time Property values 312 70,1 68,5 58,8 68,0 60,5 432 42,6 62,0 62,3 68,9 69,8 576 39,5 45,4 36,7 43,7 47,4 696 39,0 40,3 35,4 26,0 35,1 744 31,2 32,4 34,3 32,4 31,8 840 36,9 29,6 18,9 26,2 30,1 888 32,5 27,5 58,9 19,4 37,7 Times 432 to 840 selected F = 0,529 : F(0,95, 3, 20) = 3,062	Temperature 165 Time Property values 528 70,9 56,5 70,9 74,5 65,6 840 62,2 46,6 46,0 57,4 48,8 1176 9,1 39,7 42,5 45,6 54,4 1274 33,0 33,1 37,6 54,9 39,2 1344 32,7 38,8 33,1 33,9 34,8 1512 23,4 31,7 32,5 25,7 25,8 1680 21,6 26,0 25,6 21,2 25,8 1848 21,6 22,1 25,8 20,9 19,6 Times 528 to 1 848 selected F = 0,278 : F(0,95, 6, 32) = 2,532
Temperature 150 Time Property values 984 83,4 83,4 82,6 81,3 82,6 1680 71,0 71,8 74,8 71,0 68,8 2160 49,8 54,2 54,2 48,6 43,6 2304 52,4 50,1 47,1 37,5 42,4 2685 29,6 37,4 34,1 39,0 35,3 3360 39,5 37,8 27,8 36,3 26,9 Times 1 680 to 2 685 selected F = 0,342 : F(0,95, 2, 16) = 3,526 Did not cross the end point line: extrapolation 0,140	Temperature 135 Time Property values 3216 45,2 71,0 73,6 72,3 4728 49,9 70,6 66,7 63,5 59,2 5265 30,5 33,7 49,1 50,2 55,3 6072 35,4 37,7 37,7 37,3 39,0 7440 16,1 17,6 19,4 20,9 17,4 7752 21,3 20,9 20,2 21,6 18,9 8088 19,7 18,9 18,9 18,5 18,5 Times 4 728 to 7 440 selected F = 2,126 : F(0,95, 2, 16) = 3,526
End-point = 30	

Data file n3.dst: file generated by Entry.bas program.

4	56,5	82,6	55.3
8	70,9	81,3	6072
5	74,5	82,6	5
180	65,6	1680	35.4
7	840	5	37.7
312	5	71,0	37.7
5	62,2	71,8	37.3
70,1	46,6	74,8	39.0
68,5	46,0	71,0	7440
58,8	57,4	68,8	5
68,0	48,8	2160	16.1
60,5	1176	5	17.6
432	5	49,8	19.4
5	9,1	54,2	20.9
42,6	39,7	54,2	17.4
62,0	42,5	48,6	7752
62,3	45,6	43,6	5
68,9	54,4	2304	21.3
69,8	1274	5	20.9
576	5	52,4	20.2
5	33,0	50,1	21.6
39,5	33,1	47,1	18.9
45,4	37,6	37,5	8088
36,7	54,9	42,4	5
43,7	39,2	2685	19.7
47,4	1344	5	18.9
696	5	29,6	18.9
5	32,7	37,4	18.5
39,0	38,8	34,1	18.5
40,3	33,1	39,0	30
35,4	33,9	35,3	
26,0	34,8	3360	
35,1	1512	5	
744	5	39,5	
5	23,4	37,8	
31,2	31,7	27,8	
32,4	32,5	36,3	
34,3	25,7	26,9	
32,4	25,8	135	
31,8	1680	7	
840	5	3216	
5	21,6	4	
36,9	26,0	45,2	
29,6	25,6	71,0	
18,9	21,2	73,6	
26,2	25,8	72,3	
30,1	1848	4728	
888	5	5	
5	21,6	49,9	
32,5	22,1	70,6	
27,5	25,8	66,7	
58,9	20,9	63,5	
19,4	19,6	59,2	
37,7	150	5265	
165	6	5	
8	984	30,5	
528	5	33,7	
5	83,4	49,1	
70,9	83,4	50,2	

E.4 Output files and graph

The 216-3 program generates two output files. The first (file ending ".rep") is a formatted statement of the input data and the report in the format required by the standard.

The second (file ending ".int") comprises the intermediate data required by IEC 60216-5 [3] for calculation of the RTE, in the sequence specified by IEC 60216-5, readable by the computer program of that standard.

E.4.1 Thermal endurance graph

The thermal endurance graph is presented in a graphics format which may be copied into the Windows clipboard (see below). The graph may then be imported into another Windows program (e.g. a word processor) in the usual way. Graphs for materials having different ageing properties are produced with compatible temperature scales, each being in effect a "window" of fixed width in an infinitely long reciprocal absolute temperature scale.

E.4.2 Copying graphs to word processor reports

If it is required to include the graphs in a report, the word processor program should be started before the 216-3 program, which should be set up to run in a DOS window. Although the graphs are displayed full screen, the change from and back to a DOS (text) window is automatic.

When the graph is displayed, it is copied to the Windows clipboard by pressing **Print Screen** or **ALT + Print Screen** key (**ALT + Print Screen** copies the active window: **Print Screen** copies the active screen). The word processor may then be brought to the screen by pressing **ALT + Esc** or **ALT + Tab**, if necessary, repeatedly. The transfer is then completed by means of the menu function **Edit / Paste Special / Device Independent Bitmap**. Do not use the **Control + V** shortcut, as this will insert the graph in an inconvenient format (changing this is a very tedious process). The graph may be edited for size and position in the usual way. Unwanted material can generally be removed with the **crop** function of the drawing toolbar.

Return to the 216-3 program is then carried out from either the 216-3 label on the Windows taskbar or by (repeatedly) pressing **ALT + Tab** or **ALT + Esc**.

The report file *.rep may be imported directly into a word processor report and edited or formatted in the usual way.

Bibliography

- [1] SAW, J.G., Estimation of the Normal Population Parameters given a Singly Censored Sample, *Biometrika* 46, 150, 1959
 - [2] PRESS, W.H. *et al.*, *Numerical Recipes, FORTRAN Version*, Cambridge University Press, Cambridge 1989
 - [3] IEC 60216-5, *Electrical insulating materials – Thermal endurance properties – Part 5: Determination of relative thermal endurance index (RTE) of an insulating material*
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